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VERTICES IN A SCALAR-GRAVITY SYSTEM IN THE TELEPARALLEL GRAVITY THEORY

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Abstract

We consider a scalar field interacting gravitationally in teleparallel gravity theory. In this theory, the gravity is defined as a translational gauge field and thus gravitational interaction can be introduced through a minimal coupling prescription as in standard gauge theory. In this paper we consider scalar-gravity interacting terms of the scalar-gravity Lagrangian and derive the corresponding vertices by writing the tetrad field h_a^μ as $\delta_a^\mu + b_a^\mu$. We obtain six different vertices: one-gravity, two-gravity, etc. up to six-gravity vertices.

Keywords: Teleparallel gravity, scalar field, scalar-gravity vertices.

Abstrak

Kami meninjau medan skalar yang berinteraksi secara gravitasional dalam teori gravitasi teleparalel. Dalam teori ini medan gravitasi didefinisikan sebagai medan gauge translasi sehingga interaksi gravitasi dapat dibangun melalui preskripsi kopling minimal sebagaimana dalam teori gauge standar. Dalam makalah ini kami meninjau suku-suku interaksi gravitasi skalar pada Lagrangian untuk sistem medan gravitasi skalar dan menurunkan verteks-verteks yang terkait melalui pengungkapan medan tetrad h_a^μ sebagai $\delta_a^\mu + b_a^\mu$. Kami peroleh enam verteks berbeda: verteks satu gravitasi, dua gravitasi, dan seterusnya sampai verteks enam gravitasi.

Kata Kunci: Gravitasi teleparalel, medan skalar, verteks gravitasi skalar.

1. Pendahuluan

The gravitational interaction was founded long ago by Isaac Newton. Gravitational interaction describes attraction between two masses matter which called gravitational force. The field associate with these interaction is gravitational field and its known to be possessed by masses matter.

In the early of 20th century, Albert Einstein established the special theory of relativity, shortly known as special relativity (SR). Special relativity provides the notion that light will have the same speed regardless seen from any observer. Soon, this theory lead Einstein to predict the effect of SR into Newton's theory of gravitation. Then in 1915 Einstein published the general theory of relativity.
The general theory of relativity provides the notion that gravity is related to space-time. Gravity curves the space-time so objects will moves on a curved path [1]. This "new" gravitational theory lead to the more wide application of gravitational theory in the future, including GPS.
In the same era, the theory that tries to understand the behaviour of small objects (or particle) emerges, known as the quantum theory. Then, because subatomic particle travel with speed close to light speed (c = 3 x 10^8 m/s), the special theory of relativity is considered to play a role in quantum mechanics. This two theories forms quantum field theory (QFT).
There are four interactions in physics: weak, strong, gravitational, and electromagnetic. The weak, strong, and electromagnetic interaction are described by gauge field theory, a theory which has an important role in maintaining symmetry properties in QFT. Therefore, only gravitational interaction that originally can't be described using gauge theory. Attempts to using gauge theory to described gravitational interaction have been done by H. Weyl and A. Einstein [2]. Although their attempt were not successful, they have laid a foundation for a gauge theory for gravitation, named as teleparallel gravity.
In teleparallel gravity (TG) or tele-equivalent general relativity (TEGR) the gravity is defined as a translational gauge field B^μ [2],[3]. It is a translational gauge field because the corresponding generators $\partial_a \cong \partial/\partial x^a$ are the generators of translation transformations, in this case translation transformations in tangent space of a four dimensional curved space-time. In the above, the Latin indices a correspond to Minkowski or tangent while the Greek indices relate to curved space-time. Thus we can write $B^\mu = B_a{}^\mu \partial^a$ where $B_a{}^\mu$ are the components of the gauge field defined in local bases ∂^a. $B_a{}^\mu$ are defined to have a relationship with tetrad fields in Einstein general relativity.
The field strength of the translational gauge field is defined just like in the standard gauge theory. In fact, it is equivalent to the Weitzenbock torsion, the torsion defined from the curvature less Weitzenbock connection. Its explicit form, in terms of the tetrad and spin connection, is

$$\overset{\bullet}{T}{}^a{}_{\nu\mu} = \partial_\nu h^a{}_\mu - \partial_\mu h^a{}_\nu . \tag{1}$$

Note that in general there are spin connection terms, however in TG this connection is to vanish [2], resulting that the torsion is just like the field strength for an abelian gauge field such as the electromagnetic field. The Lagrangian of the gravitational field is defined as quadratic in the field strength or the Weitzenbock torsion [2],[3]:

$$\frac{2k}{h} \overset{\bullet}{\mathcal{L}}{}_{grav} = \frac{1}{4} \overset{\bullet}{T}{}^\rho{}_{\mu\nu} \overset{\bullet}{T}{}_\rho{}^{\mu\nu} + \frac{1}{2} \overset{\bullet}{T}{}^\rho{}_{\mu\nu} \overset{\bullet}{T}{}^{\nu\mu}{}_\rho - \overset{\bullet}{T}{}^\rho{}_{\mu\rho} \overset{\bullet}{T}{}^{\nu\mu}{}_\nu . \tag{2}$$

In the above, $h = \det(h_a{}^\mu)$. However uunlike in the standard gauge theory, we have in the above expression, two extra terms, i.e. the last two terms. This is due to one extra number of space-time indices for the torsion compared to that for the field strength in standard gauge theory.

Since the gravitational field in TG is a gauge field, matter fields that interact with gravity can be represented by Lagrangians obtained from applying minimal coupling prescription to the Lagrangian of the corresponding free matter fields. Such Lagrangians have been derived [2],[3]. Field equations for some space-time have been solved for certain conditions [4],[5],[6]. In this paper we will consider the Lagrangian for a scalar (scalar) field that interacts with gravity and find out the corresponding vertices.
The organization of the paper is the following. In the next section we will review the scalar-gravity system in TG theory. Here we will show the Lagrangian of the system. This Lagrangian will then be utilized in Section 3 to obtain vertices. Section 4 is devoted for conclusions.

2. Lagrangian of Scalar-Gravity System

The Lagrangian of a scalar-gravity system, according to gauge theory, consists of the Lagrangian of gravity plus the Lagrangian of a free scalar field after replacing the space-time partial derivative in the later Lagrangian by the covariant one. In this case the covariant derivative is of the form

$$\dot{\mathcal{D}}_a \equiv h_a^\mu \underset{\mu}{\dot{\mathcal{D}}} = h_a^\mu \left(\partial_\mu + \tfrac{i}{2} \dot{K}^{ab}{}_{\mu}\, S_{ab} \right) \tag{3}$$

In the above, $\dot{K}^{ab}{}_{\mu}$is the Weitzenbock contortion and S_{ab} is the generator of the Lorentz group. The free scalar field Lagrangian is defined in Minkowski or tangent space as the following

$$\mathcal{L}_\phi = \tfrac{1}{2}\eta^{ab}\partial_a\phi\,\partial_b\phi - m^2\phi^2. \tag{4}$$

Thus insertion of a gravitational effect in TG to a scalar field is equivalent to changing partial derivatives in Minkowski space ∂_a by the covariant one $\dot{D}_a$(also in Minkowski space):

$$\dot{\mathcal{L}}_{\phi} = \frac{h}{2}\left(g^{\mu\nu}\, \dot{D}_\mu\, \phi\, \dot{D}_\nu\, \phi - m^2\phi^2 \right). \tag{5}$$

The complete system of a scalar-gravity field is described by the Lagrangian (2) and (5):

$$\begin{aligned} \dot{\mathcal{L}} &= \tfrac{h}{2}\left(g^{\mu\nu}\, \dot{D}_\mu\, \phi\, \dot{D}_\nu\, \phi - m^2\phi^2 \right) + \\ &\tfrac{h}{2k}\left(\tfrac{1}{4}\dot{T}^{\rho}{}_{\mu\nu}\, \dot{T}_{\rho}{}^{\mu\nu} + \tfrac{1}{2}\dot{T}^{\rho}{}_{\mu\nu}\, \dot{T}^{\nu\mu}{}_{\rho} - \dot{T}^{\rho}{}_{\mu\rho}\, \dot{T}^{\nu\mu}{}_{\nu} \right). \end{aligned} \tag{6}$$

In the above, $k{=}8\pi G/c^4$ where G is the gravitational constant.

3. Vertices

The dynamics of a free scalar field ϕ is described by the Klein-Gordon equation which is a linear differential equation. The corresponding Lagrangian has a bilinear form whose

function in between the fields $\phi\ (x)$ and $\phi\ (y)$ describes inverse of the Green function, or the inverse propagator [7],[8]. When the field interacts, the gauge field is introduced resulting in new terms in the Lagrangian, the interaction terms. These terms are non-bilinear, consist of two scalar field and at least one gauge field, and define vertices of the theory. Scattering of the field perturbatively is expressed by Feynman diagrams, each containing vertices, propagators, and of course the fields [7],[8].
For a scalar field interacting gravitationally through teleparallel gravity as described by equation (6) the vertices come from the non-bilinear terms within the term containing covariant derivatives, thus equation (5). Since scalar fields belong to the null representation $S_{ab}\phi = 0$, the equation (5) can be written as

$$\mathcal{L}_\phi = \frac{h}{2}\left[\eta^{ab}h_a^\mu h_b^\nu(\partial_\mu\phi)(\partial_\nu\phi) - m^2\phi^2\right] \tag{7}$$

Because tetrad fields ${h_a}^\mu$ represent gravitational fields, all terms in (7) are non-bilinear. Accordingly, to develop perturbation theory for scalar fields interacting gravitationally, that is to write the above Lagrangian as the Lagrangian for free field (equation (4)) plus an interacting Lagrangian, one must write

$$h_a^\mu = \delta_a^\mu + b_a^\mu. \tag{8}$$

Here the field ${b_a}^\mu$ replaces ${h_a}^\mu$ to represent the gravitational field.
As h has the form of a fourth power of ${h_a}^\mu$, equation (8) gives the form

$$h = 1 + 4(b) + 12(bb) + 24(bbb) + 24(bbbb). \tag{9}$$

The coefficients in (9) describe the number of terms for different powers of b.
The last four terms in equation (9) and the mass term of the Lagrangian define the following vertices (solid lines represent scalar fields while wavy lines represent gravitational fields $({b_a}^\mu)$):

We have four one-gravity vertices with different ${b_a}^\mu({b_0}^0,\ {b_1}^1,\ {b_2}^2,\ {b_3}^3)$; twelve two-gravity vertices with the forms of ${b_a}^{\mu=a}{b_b}^{\nu=b}$and $-{b_a}^{\mu=b}{b_b}^{\nu=a}$ with all possible but different values of a,b; twenty four three-gravity vertices with the forms of ${b_a}^\mu {b_b}^\nu {b_c}^\alpha$ with all possible but different values of a, b, c and permutation of μ=a, ν=b, α=c (with negative sign for odd permutation); and twenty four four-gravity vertices with the forms of ${b_a}^\mu {b_b}^\nu {b_c}^\alpha {b_d}^\beta$ with all possible but different values of a, b, c, d and permutation of μ=a, ν=b, α=c, β=d (with negative sign for odd permutation).
Now let us consider the first term of equation (7). It has the factor of $h(\eta^{ab}h_a^\mu h_b^\nu)$. Its explicit form consists of one b-free term describing the kinetic term of the Lagrangian and a number of b-terms, first power to sixth power of b. To express the corresponding vertices let us write

$$\begin{aligned}\phi(x) &= \textstyle\int dk\ \phi(k)\exp(ikx)\\ &= \textstyle\int dp\ \phi(p)\exp(ipx).\end{aligned} \tag{10}$$

Accordingly, the first term of equation (7) in momentum space has the form

$$h\eta^{ef}(k_e p_f + k\cdot b_e p_f + p\cdot b_e k_f + k\cdot b_e p\cdot b_f). \tag{11}$$

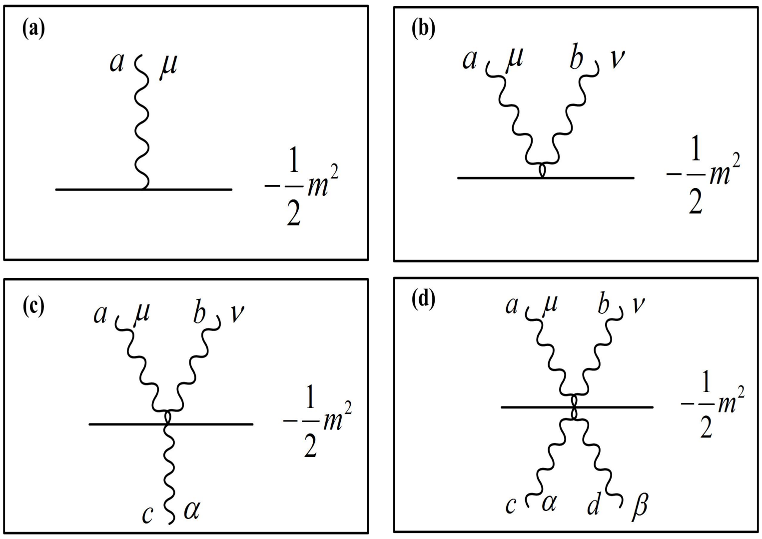

Figure 1. (a)One - Gravity Vertex, (b) Two-Gravity Vertex, (c)Three-Gravity Vertex, (d)Four-Gravity Vertex

Note that the above dot product is defined for space-time indices. Recalling equation (9), equation (11) leads to the following vertices in momentum space. The first term in equation (11) gives a kinetic term and vertices with the same diagrams as in figure 1 but by replacing $-\frac{1}{2}m^2 \to -\frac{1}{2}\eta^{ab}k_a p_b$.

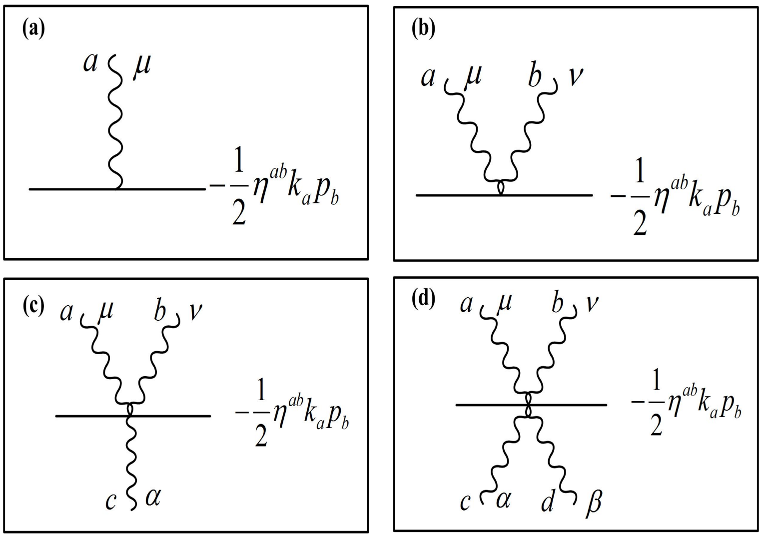

Figure 2. (a)One - Gravity Vertex, (b) Two-Gravity Vertex, (c)Three-Gravity Vertex, (d)Four-Gravity Vertex

Vertices corresponding to the second and third terms of equation (11) can be obtained from diagrams in figure 1 by adding one extra $b_e{}^{\sigma}$-field and by changing

$$-\tfrac{1}{2}m^2 \to -\tfrac{1}{2}\eta^{ef}(k_\sigma p_f + p_\sigma k_f).$$

(k and p are the momenta of scalar fields):

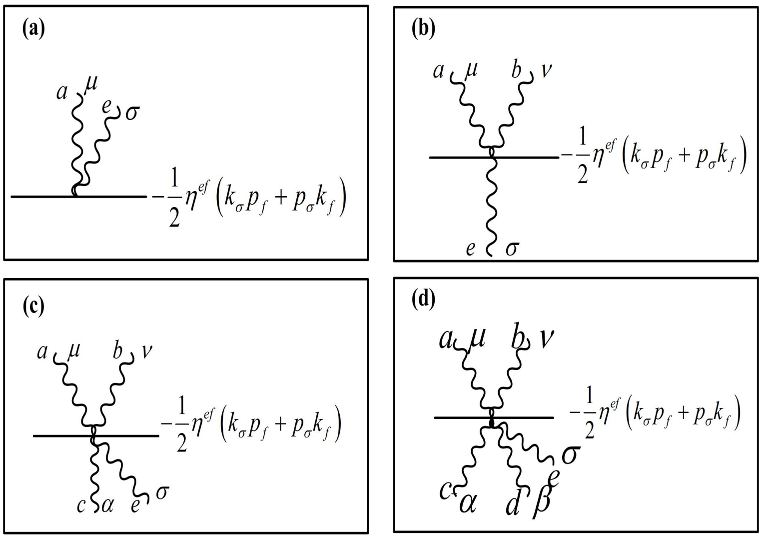

Figure 3. (a)Two - Gravity Vertex, (b) Three-Gravity Vertex, (c)Four-Gravity Vertex, (d)Five-Gravity Vertex

Vertices corresponding to the last term of equation (11) can be obtained from diagrams in Figure 1 by adding two extra ${b_e}^\sigma$ and ${b_f}^\rho$ fields and by replacing $-\frac{1}{2}m^2 \to -\frac{1}{2}\eta^{ef}k\cdot b_e p\cdot b_f$.

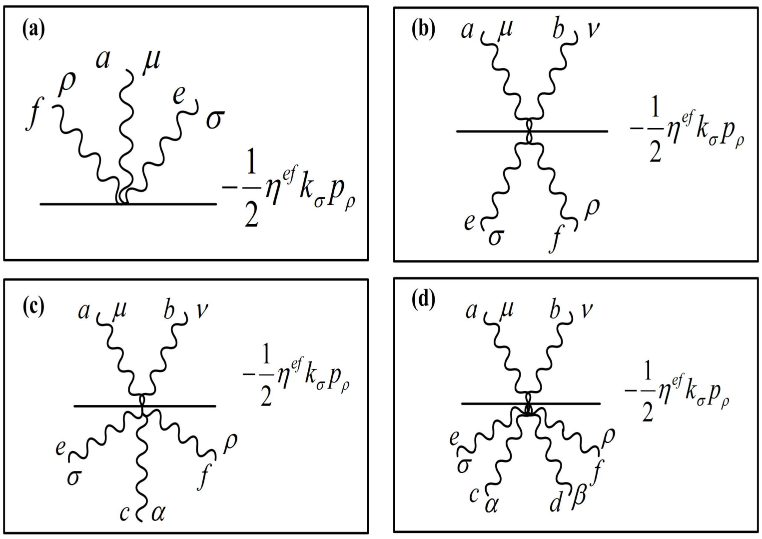

Figure 4. (a)Three - Gravity Vertex, (b) Four-Gravity Vertex, (c)Five-Gravity Vertex, (d)Six-Gravity Vertex

4. Conclusions

We have derived scalar-gravity vertices of the scalar field interacting gravitationally in TG. There are six main different vertices, namely, one, two, three, four, five, and six gravity vertices. Figure 1.a and figure 2.a constitute one gravity vertices, figure 1.b, figure 2.b, and figure 3.a give two gravity vertices, while figure 1.c, figure 2.c, figure 3.b, and figure

4.a give three gravity vertices. We have four gravity vertices from figure 1.d, figure 2.d, figure 3.c, and figure 4.b, five gravity vertices from figure 3.d, and figure 4.c, and finally six gravity vertices from figure 4.d. Figures 5 below summarizes the vertices.

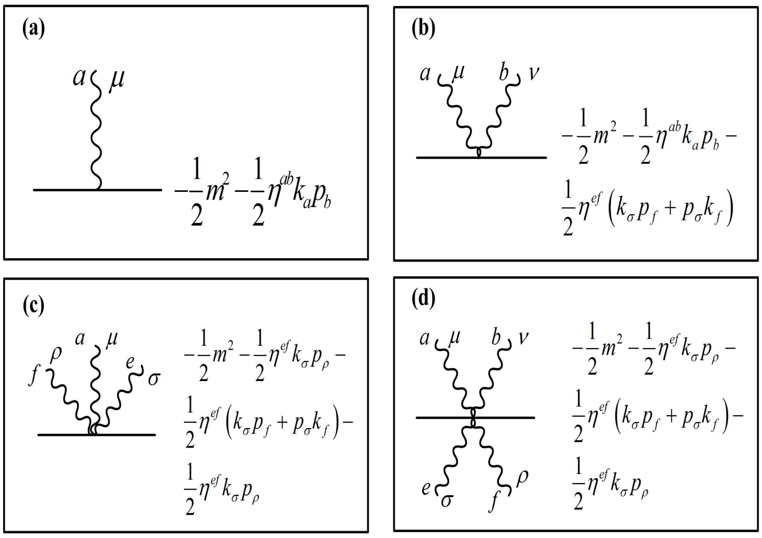

Figure 5. (a)One - Gravity Vertex, (b) Two-Gravity Vertex, (c)Three-Gravity Vertex, (d)Four-Gravity Vertex, (e)Five-Gravity Vertex, (f)Six-Gravity Vertex

The above vertices will be the basis for Feynman diagrams of scalar fields scattered gravitationally. In computing the scattering we need to know propagators for both scalar dan gravitational fields. This will be considered elsewhere.

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