BARRIER OPTION

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Abstrak:

Barrier option merupakan salah satu bentuk option yang akan mulai berlaku atau akan dibatalkan apabila harga aset menyentuh tingkat barrier tertentu. Terdapat 4 tipe barrier option yaitu up and in option, up and out option, down and in option, dan down and out option. Nilai barrier option dapat dihitung dengan menggunakan beberapa metode, di antaranya adalah model Black-Scholes, Monte Carlo Simulation dan teknik Binomial Tree. Barrier option dapat digunakan sebagai salah satu alternatif hedging, walaupun dalam pelaksanaannya lebih sulit dibandingkan dengan option biasa.

Barrier options are the popular form of exotic options. Exotic options refer to non-standard option, which emerge due to the development in financial and option technology, the increased focus on risk management and the increased focus on the inadequacies of existing risk-management tools. Barrier options are becoming more common in the over the counter markets as alternative hedging tools since they provide cheaper price than plain European options. Barrier option is non-standard option because it has more complicated payoffs than the standard European and American option and has different activation or expiration mechanism with standard option.

The Development of Barrier Option as Part of Exotic Option

Barrier option is one type of exotic option, so that the growth of barrier option can be seen from the growth of exotic option. The emergence of exotic option reflects the growth of financial derivatives generally and the increased demand for highly customized risk-management or hedging structure by investor in capital market.

The development of exotic option can be seen as a process of risk management evolution, which include the following factor (*Das, 1996*):

- ***Uncertainty and the volatility in asset market***

Since early 1970's, financial market volatility has increased. It followed by increasing of volatility in foreign exchange market and in interest rate in the late of 1970's and early 1980's. The volatility and uncertainty make some problem in risk management, so that the investor and borrower needed instruments, which could hedge this risk. It tended to create the innovation in risk management instruments.

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- ***Increased focus on financial risk management***

  The development of innovation in risk management increased the focus on financial risk management. The volatility and uncertainty have led to this increased focus.

- ***Demand for highly customized risk-reward profiles***

  The increased focus on financial risk management created demand for highly customized risk reward profiles. It tended to increase the trend towards active management of financial risk and the investor demand for yield of return improvement at the given risk.

- ***Development option pricing and hedging technology***

  The development in option pricing and technology facilitated the demand of the demand of the newer and more innovative risk management instruments. The financial technology evolution accelerated the development of computer hardware and software which applied the application of mathematical techniques for pricing and management of derivatives products.

- ***The politics of risk management***

  The politics of risk management referred to the adoption by entities of appropriate policies and procedures, which related to the measurement, management and evaluation of financial risk. The evidence showed that entities has been fail to adopt risk management and in positioning/speculative strategy. The political nature of the risk management has created a good prospect for the development of innovative risk management products.

The evolution of risk management consists of 3 generation products. The barrier option as part of exotic option is included in the third generation product, which focus on innovative option structure.
There are some advantages of barrier options:

1. The lower premium relative to standard options.
   This lower premium reflects the possibility that the option will be extinguished or not activated. This premium depends on the relationship between the spot and forward price level, barrier level and the time to maturity of the option.
2. The ability to create highly structured forms of protection in line with asset-price expectations.

Types and Payoffs of Barrier Option

Technically, barrier options are a class of path-dependent options. These are a class of option transactions, which entail a mechanism where the option contract is activated and deactivated as a function of the level of the underlying asset price. In other word, barrier options can be defined as options that exist or cancel if the price of underlying asset reach a certain barrier price. Generally, there are 4 type of barrier options:

1. *Up and out options*

These options will cease to exist if at any time during the life of the option, price of underlying assets hits the certain price, which is above the spot price at origination. These options will expire worthless.

2. *Up and in options*

Up and in options will start to active as plain option if the barrier, which is above the current spot price, is hit. If the asset price never reached the barrier, the options never come into existence and would then expire with no value at expiration, even the spot price is higher than the strike price (or in the money condition).

3. *Down and out options*

These are standard options that will terminate before expiration or expire worthless if the underlying asset price reaches the barrier, which is below the initial price of underlying asset.

4. *Down and in options*

Down and in options are the regular options that come to existence only when the barrier is reached. In these options, barrier level is lower than the current price of underlying asset.

All terminology above could be used in both call and put option. Hence, there are eight types in barrier option, i.e. *down and in call, up and in call, down and out call, up and out call, down and in put, up and in put, down and out put,* and *up and out put.* The payoff each type is:

Down and in Call	Max (0,S-X) if one $S \geq B$, else 0
Up and in Call	Max (0,S-X) if one $S \leq B$, else 0
Down and out Call	Max (0,S-X) if one $S > B$, else R
Up and out Call	Max (0,S-X) if one $S < B$, else R
Down and in Put	Max (0,X-S) if one $S \geq B$, else 0
Up and in Put	Max (0,X-S) if one $S \leq B$, else 0
Down and out Put	Max (0,X-S) if one $S > B$, else R
Up and out Put	Max (0,X-S) if one $S < B$, else R

where: B = Barrier, S = Spot Price, X = Strike Price, R = Rebate
Rebate is sum of money which investor received if the spot price of underlying asset is never touched the barrier level (in the case knock-in option) and if the price is touched the barrier level (in the case knockout option).

For example, one down and out call option is giving the right to buy stock at price of \$30 at maturity for one month and the barrier level is \$25 (is below the exercise price and in the out of the money condition). This option has a premium of \$5. At maturity, there are two possibilities outcome:

- If the spot rate never touch the barrier level during the life of option, then

- if spot rate above $30, option will be exercised and the payoff would be the difference between spot rate and the exercise price minus premium
- if spot rate below $30, option will not be exercised

- if the spot rate reached the barrier level, option cannot be exercise since the contract has been terminated. The payoff for this condition is rebate.

Pricing Formula for Barrier Option

There are some approach to determine the pricing formula pf barrier option, such as the Black-Scholes option pricing model, Monte-Carlo simulation and binomial option pricing model. The Black-Scholes model assumes that distribution of price of underlying asset at expiration could be described by a lognormal distribution curve centered at the forward rate of the underlying asset at expiration, the width was determined by the volatility of price of the underlying asset *(Hudson, 1991)*. The pricing formula for barrier option is more complex than the standard Black-Scholes formula, since the pricing for barrier option must be evaluate the probability distribution of the condition of the underlying asset price either hit or not hit the barrier.

Monte Carlo simulation is another approach for pricing barrier option. This technique involves a generation of random numbers to solve deterministic problems and simulating possible future movement in price of underlying asset. The disadvantage of Monte Carlo simulation is that time consuming due to the high monitoring frequency of the underlying asset price *(Kat and Verdonk, 1995)*. This approach makes the difficulty in calculating the sensitivity of an option and solving the relevant value of hedging information.

The recent development of the pricing barrier option is using a binomial tree. Binomial tree technique chooses the representative values for the underlying asset at each node of the tree and calculates the value of the option starting at the end of tree and working backward. The problem of using this method is the convergence of the option value to the correct value as the number of time steps is increased tends to be slow *(Hull, 1997)*. TO overcome this problem, there are two alternative ways, i.e. (1) arranging the geometry of the tree so that nodes always lies on the barriers and (2) using the interpolation scheme to adjust for fact that the barrier being assumed by the binomial tree is different from the true barrier.

The value of each type of barrier options are presented by the following formulas *(Hull, 1997)*:

- **Call Option**
  - ***Up and In***

    When the barrier level (B) is greater than or equal to exercise price (X), the value of an up and in call option is

$$c_{ui} = SN(x_1)e^{-qT} - Xe^{-rT}N(x_1 - \sigma\sqrt{T}) - Se^{-qT}(B/S)^{2\lambda}[N(-y) - N(-y_1)]$$
$$+ Xe^{-rT}(B/S)^{2\lambda-2}\left[N(-y+\sigma\sqrt{T}) - N(-y_1+\sigma\sqrt{T})\right]$$

where,

$$x_1 = \frac{ln(S/B)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$y_1 = \frac{ln(B/S)}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

$$\lambda = \frac{r - q + \sigma^2/2}{\sigma^2}$$

$$y = \frac{ln[H^2/(SX)]}{\sigma\sqrt{T}} + \lambda\sigma\sqrt{T}$$

When B is below or equal to X, the value of an up and in call option equals to the value of a regular call option.

- ***Up and out***

  When B is above or equal to X, the value of an up and out call option equals to the value of a regular call option minus the value of an up and in call option:

$$c_{uo} = c - c_{ui}$$

When B is below or equal to X, the value of an up and out call option equals to zero.

- ***Down and out***

  When B is greater or equal to X, the value of a down and out call option is:

$$c_{do} = SN(x_1)e^{-qT} - Xe^{-rT}N(x_1 - \sigma\sqrt{T}) - Se^{-qT}(B/S)^{2\lambda}N(y_1)$$
$$+ Xe^{-rT}(B/S)^{2\lambda-2}N(y_1 - \sigma\sqrt{T})$$

When B is below or equal X, the value of a down and out call option equals to the value of regular call option minus the value of down and in call option, since the value of regular option is equal to the value of up an in option plus the value of up and out option:

$$c_{do} = c - c_{di}$$

- ***Down and in***
  When B is greater than or equal to X, the value of a down and in call option equals to the value of a regular call option minus the value of a down and out call option:

$$c_{di} = c - c_{do}$$

When B is below or equal X, the value of a down and in call option is:

$$c_{di} = Se^{-qT}(B/S)^{2\lambda}N(y) - Xe^{-rT}(B/S)^{2\lambda-2}N(y - \sigma\sqrt{T})$$

- **Put Option**
  - ***Up and in***
    When B is above or equal to X, the value of an up and in put option is:

$$p_{ui} = -Se^{-qT}(B/S)^{2\lambda}N(-y) + Xe^{-rT}(B/S)^{2\lambda-2}N(-y + \sigma\sqrt{T})$$

When B is below or equal to X, the value of an up and in put option equals to the value of a regular put option minus the value of an up and out put option:

$$p_{ui} = p - p_{uo}$$

- ***Up and out***
  When B is above or equal to X, the value of an up and in put option equals to the value of a regular put option minus the value of an up and in put option:

$$p_{uo} = p - p_{ui}$$

When B is below or equal to X, the value of an up and out put option is:

$$p_{uo} = -SN(-x_1)e^{-qT} + Xe^{-rT}N\left(-x_1 - \sigma\sqrt{T}\right) + Se^{-qT}(B/S)^{2\lambda}N(-y_1)$$
$$-Xe^{-rT}(B/S)^{2\lambda-2}N\left(-y_1 - \sigma\sqrt{T}\right)$$

- ***Down and in***

  When B is above or equal to X, the value of a down and in put option equals to the value of a regular put option.

  When B is less than or equal to X, the value of a down and out put option is:

$$p_{di} = -SN(-x_1)e^{-qT} + Xe^{-rT}N\left(-x_1 + \sigma\sqrt{T}\right) + Se^{-qT}(B/S)^{2\lambda}[N(y) - N(y_1)]$$
$$-Xe^{-rT}(B/S)^{2\lambda-2}\left[N\left(y - \sigma\sqrt{T}\right) - N\left(y_1 - \sigma\sqrt{T}\right)\right]$$

- ***Down and out***

  When B is higher than or equal to X, the value of a down and out put option equals to zero.

  When B is lower than or equal to X, the value of a down and out put option equals to the value of a regular put option minus the value of a down and out put option:

$$p_{do} = p - p_{di}$$

Hedging Strategy with Barrier Options

Barrier options have a volatile risk profile. By combining barrier option with the underlying stock or by combining barrier options into portfolios, the portfolio manager can create positions with exactly the desired risk exposures. A hedger needs to know how a given position responds to changing parameters to understand a hedge completely and to create more effective hedges. Barrier option is more difficult to hedge than the regular option due to the delta of the options is discontinuous at the barrier. There are four key parameter that determine sensitivity the price of an option to the stock price, the standard deviation of stock's return, the interest rate, and the time remaining until maturity:

- ***Delta***

  Delta measures the sensitivity of the option's price to changing the price of underlying asset. Delta for barrier option is typically positive (for a call option) but it becomes negative as it approaches the barrier
- ***Gamma***

  Gamma is the rate of change of the delta with respect to the change of price of the underlying asset. Barrier options demonstrates very high gammas and deltas when the asset price is trading around the out or in expiration.

- ***Vega***
  Vega measures the sensitivity of the price of the option to changing the volatility of the underlying asset. For knockout options, the increase in volatility can reduce the option premium since the higher volatility the greater the possibility to reach the barrier level and the option expire worthless.

- ***Theta***
  Theta is the rates of change of the price of the option with respect to passage of time. The tendency for option prices to change due to the passage of time refers to time decay. In barrier option is extinguished or activated.

There are two approaches to hedging barrier option portfolio:

- First approach is investors replicate barrier options by trading in a series of options. For example, the trader replicate a knockout call option by simultaneously buying a standard call option at the same exercise price as the knockout call option and selling a standard put option with the lowered exercise price. The difference between both exercises price equals to knockout premium.
- Second approach is investors use the traditional replication strategy, which beside holding the options they take position in the underlying asset. In this approach, investors could manage the portfolio, except when the underlying is close to the out- or in- strike. Therefore, the investor must consider to the critical hedging event, which is whether delta neutrality or the relevant replication position can be maintained by reversing the hedge instaneously if an out- or in- strike event occurs.

Summary

Barrier option becomes more common in over the counter market since the late 1970s. Barrier option is one type of exotic option which is defined as options that exist or cancel if the price of underlying asset hit the certain barrier level. There are 4 main type of this option, i.e. up and in option, up and out option, down and in option, and down and out option. To value the barrier option, some methods could be used, such as Black-Scholes model, Monte Carlo simulation and binomial tree technique. Each method has the problem in valuing the option. By combining the barrier option to portfolio, the risk exposure could be hedged. However, hedging barrier option is more difficult than the regular option.

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