OPTIMIZATION OF LOCATION MODEL OF CAPACITATED NETWORK

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Abstract

This study concerns with the research work on Location Model of Public Service Obligation State-Owned Company (PSO-SOC). Its aim is to develop an approach of making location decision on distribution facilities of such company, in which capacities of facilities are limited (capacitated facilities) and it deals with single commodity. Heuristic solution is proposed to modify established Add Algorithm, which is designed for uncapacitated facilities. The Network Representation is used to represent original problem of Location Model. An example is provided to illustrate the proposed step-wise of solving the model.

Keywords: location model, Add Algorithm, Network Representation.

Abstrak

Penelitian ini terkait dengan Model Lokasi *Public Service Obligation State-Owned Company* (PSO-SOC). Tujuannya adalah untuk mengembangkan suatu pendekatan untuk pengambilan keputusan lokasi pada fasilitas-fasilitas distribusi perusahaan tersebut, dengan kapasitas fasilitas yang terbatas (fasilitas dengan kapasitas yang dilampaui) dan berhubungan dengan komoditas tunggal. Suatu solusi diusulkan untuk memodifikasi *Add Algorithm*, yang dirancang untuk fasilitas yang tidak dilampaui kapasitasnya. *Network Representation* digunakan untuk mewakili persoalan awal suatu Model Lokasi. Suatu contoh dibahas untuk menjelaskan langkah-langkah yang diusulkan untuk menyelesaikan model yang dibahas.

Kata-kata kunci: model lokasi, *Add Algorithm, Network Representation.*

INTRODUCTION

This research work is a part of a serial research on strategic design of distribution system of state-owned companies. The primary objective of the study is to develop the optimization model of determining the location of distribution facilities of a Public Service Obligation State-Owned Company (PSO-SOC). From the previous study (Soehodho and Nahry,2009), it is resumed that the location model of PSO-SOC should consider transportation cost, production cost, revenue, as well as fixed cost of facility as its basic model variables. Furthermore, demand satisfaction criteria is differentiated between subsidized demand and commercial one, whereas subsidized demands must be fully satisfied no matter possible profit the company could take and the commercial ones only could be fulfilled in case of excess plant capacity exists. This study focuses on Indonesia's PSO-SOC which deals with the production and distribution of public commodities. Such commodities are widely used in most parts of the country as the vital substance of agriculture productivity.

As a continuation of the previous work, this study is intended to develop the approach of solving location model, in which the capacities of warehouse are limited. Such warehouses are functioned as either consolidation centers or distribution centers. Hence, the real problem is how to decide whether each of set of potential warehouses should be opened or not in order to attain maximum profit in serving demand at some points of retailers from available supply at points of plants. Soehodho and Nahry (2008) proposed a physical distribution network which consists of 3 (three) stages as illustrated in Figure 1 as follows:

1. First Stage; represents links either between Plant and Consolidation Centers or between Plant and Distribution Centers.
2. Second Stage; represents links between Consolidation Centers and Distribution Centers.
3. Third Stage; represents links between Distribution Centers and Retailers.

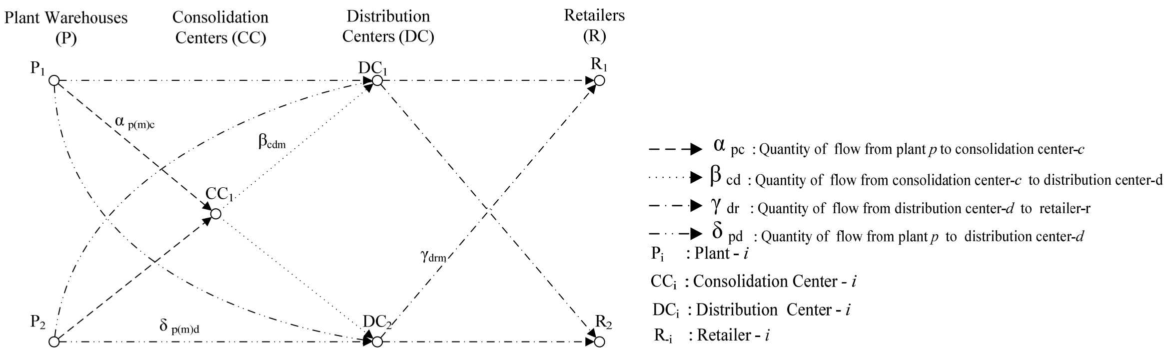

Figure 1 An Example of Physical Distribution Network

Equation (1) denotes the objective function of the proposed model. The first four fractions of such equation represent transportation cost, the fifth is associated with production cost, the sixth and the seventh represent fixed cost of facility of consolidation center and distribution center, respectively, and the last part concerns with revenue. The objective function is actually to mazimize profit of the system. Since profit is actually revenue substracted by cost, hence the objective of maximizing profit is replaced by minimizing cost, that is minimizing cost minus revenue.

The decision variables of such objective function are flows on all links of distribution network, as well as binary variables X_c and Y_d which is valued as zero or one. Zero value of X_c and Y_d illustrates that the associated facility should not be opened, whereas one shows the contrary. Equations (2) and (3) illustrate that the flow requirements in all consolidation centers, as well as distribution centers, are set as zero because those nodes are set as intermediate nodes. Equations (4) and (5) are related to demand satisfaction. Equation (4) is related to subsidized product, hence the equal sign is used in order to guarantee that such products are fully satisfied. Whereas in equation (5) less than/

equal to sign is used since commercial products should not be fully satisfied. Equations (6) and (7) guarantee that there will be no inflow and outflow in every closed facility. Equations (8), (9), and (10) show that total amount of product handled in each plant or warehouse (center) should not be more than its capacity. Equations (11) to (14) concern with non negative flow constraints and equations (15) and (16) are binary number constraints.

The location problem to be optimized in this study is modeled in the following formula:

$$\min Z\ (\alpha_{pc}, \beta_{cd}, \gamma_{dr}, \delta_{pd}) =$$

$$\sum_{p \in P}\sum_{c \in C} u_{pc} . \alpha_{pc} + \sum_{c \in C}\sum_{d \in D} v_{cd}\ \ \beta_{cd} . + \sum_{p \in P}\sum_{d \in D} z_{pd} . \delta_{pd} + \sum_{d \in D}\sum_{r \in R} w_{dr} . \gamma_{dr} + \sum_{p \in P}\left(\sum_{c \in C} \alpha_{pc} + \sum_{d \in D} \delta_{pd}\right) . \eta_p$$

$$+ \sum_{c \in C} X_c . FC_c + \sum_{d \in D} Y_d . FD_d - \sum_{d \in D}\sum_{r \in R} \gamma_{dr} . \rho_r \qquad (1)$$

subject to :

$$\sum_{p \in P} \alpha_{pc} = \sum_{d \in D} \beta_{cd}\ , \qquad \forall c \in C \qquad (2)$$

$$\sum_{c \in C} \beta_{cd} + \sum_{p \in P} \delta_{pd} = \sum_{r \in R} \gamma_{dr}\ , \qquad \forall d \in D \qquad (3)$$

$$\sum_{d \in D} \gamma_{dr} = \lambda_r\ , \qquad \forall r \in R, \forall S \qquad (4)$$

$$\sum_{d \in D} \gamma_{dr} \le \lambda_r\ , \qquad \forall r \in R, \forall C \qquad (5)$$

$$\gamma_{dr} \le Y_d . \lambda_r\ , \qquad \forall d \in D, \forall r \in R \qquad (6)$$

$$\sum_{p \in P} \alpha_{pc} \le X_c . \sum_{r \in R} \lambda_r\ , \qquad \forall c \in C \qquad (7)$$

$$\sum_{c \in C} \alpha_{pc} + \sum_{d \in D} \delta_{pd} \le Cp_p\ , \qquad \forall p \in P \qquad (8)$$

$$\sum_{c \in C} \beta_{cd} + \sum_{p \in P} \delta_{pd} \le Cd_d\ , \qquad \forall d \in D \qquad (9)$$

$$\sum_{p \in P} \alpha_{pc} \le Cc_c\ , \qquad \forall c \in C \qquad (10)$$

$$\alpha_{pc} \ge 0\ , \qquad \forall p \in P, \forall c \in C \qquad (11)$$

$$\beta_{cd} \ge 0\ , \qquad \forall c \in C, \forall d \in D \qquad (12)$$

$$\gamma_{dr} \ge 0\ , \qquad \forall d \in D, \forall r \in R \qquad (13)$$

$$\delta_{pd} \ge 0\ , \qquad \forall p \in P, \forall d \in D \qquad (14)$$

$$X_c = [0,1]\ , \qquad \forall c \in C \qquad (15)$$

$$Y_d = [0,1]\ , \qquad \forall d \in D \qquad (16)$$

with:

Subscripts:

p : indicate the Plants
c : indicate the Consolidation Centers
d : indicate the Distribution Centers
r : indicate the Retailers

Sets :

P : Set of plants
C : Set of consolidation centers
D : Set of distribution centers
R : Set of retailers
S : Set of subsidized (public) products
C : Set of commercial products

Decision Variables:

X_c = 1 if Consolidation Center - c is opened; 0 otherwise
Y_d = 1 if Distribution Center - d is opened; 0 otherwise
α_{pc} is quantity of flow from Plant p to Consolidation Center-c
β_{cd} is quantity of flow from Consolidation Center-c to Distribution Center-d
γ_{dr} is quantity of flow from Distribution Center-d to Retailer-r
δ_{pd} is quantity of flow from Plant p to Distribution Center-d

Input Parameters:

ρ_r : selling price of the product at retailer- r
u_{pc} : unit transportation cost from Plant-p to Consolidation Center-c
v_{cd} : unit transportation cost from Consolidation Center-c to Distribution Center-d
w_{dr} : unit transportation cost from Distribution Center-d to Retailer-r
z_{pd} : unit transportation cost from Plant p to Distribution Center-d
η_p : unit cost of production in plant-p
FC_c : fixed cost of facility of Consolidation Center - c
FD_d : fixed cost of facility of Distributor Center - d
λ_r : demand in Retailer-r
Cp_p : capacity of plant-p
Cd_d : capacity of DC-d
Cc_c : capacity of CC-c

THE USE OF NETWORK REPRESENTATION

Some techniques were applied to approach and solve minimization problem which is stated in previous section. In this work, Network Representation (NR) is employed in order to formulate and solve proposed model. NR is characterized by the use of diagram which is commonly used in graph theory. The proposed mathematical model, as shown in equation (1) to (16), are translated into network flow problem by adding some dummy links and nodes to represent all the components of the objective function of proposed model (Daskin, 1995).

Figure 2 denotes an example of NR of the problem of network on Fgure 1. Links between P_i - CC_i , P_i - DC_i , CC_i'- DC_i and DC_i'- R_i are designed as transportation cost links, and they represent transportation cost between two nodes. Hence, each link of transportation links is characterized by a certain unit cost of transportation.

Links between node $P_i^{'}$ and P_i are designed as production cost links. Those links represent cost of production in plant-i. Links between CC_i -CC_i' and DC_i -DC_i' are fixed cost of facility links or warehouse cost links, and those represent cost of building facility CC_i or DC_i .Links between either node R_i and R_i^S or node R_i and R_i^C are designed as revenue links. Those links represent negative revenue from selling subsidized or commercial product in retailer-*r*.

Each node of NR is valued as its Flow Requirement. Flow Requirement of node P_i' is set as capacity of plant-*i*. Flow Requirements of node R_i^S and R_i^C are set as demand of retailer-*i* on subsidized product and commercial one,respectively.Flow Requirement of the remaining nodes, those are P_i , CC_i, CC_i', DC_i , DC_i' and R_i are set as zero. It implies that those points are just designed as intermediate nodes.

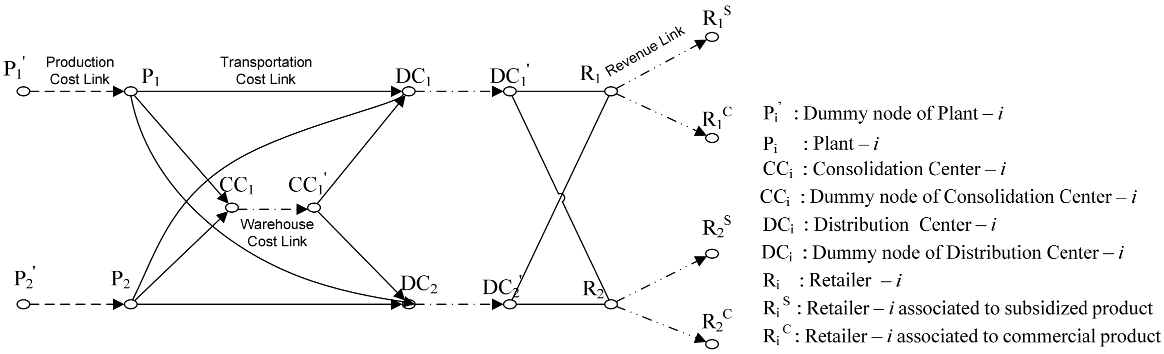

Figure 2 An Example of Network Representation

In order to ensure that the total demand is always in balance with total supply, additional subnetwork is developed called Excess Supply/Demand Subnetwork. An example of Excess Demand subnetwork is depicted in Figure 3 and Figure 4, while the one of Excess Supply is depicted in Figure 5.

In case of total demand is greater than total supply, the excess demand case is solved by permitting the system to receive additional supply from other source beyond the system. Practically, this scheme is in accordance with import policy. In order to translate such policy into NR, Import node and the associated import links are added which connecting such node to any node in NR which is permitted to receive import product (Figure 3). In the example of Figure 3, plants, consolidation centers, distribution centers, as well as retailers, are permitted to receive product and distribute them to the retailer. Surely, import product in retailer nodes do not need to be distributed any longer.

The other solution of excess demand case is by diminishing the demand at certain nodes of retailers. In such case, priorities have to be made for subsidized demand to be fully satisfied. This idea is translated into NR by adding dummy plant which functioned to act as if supply the retailer nodes through the excess demand links (Figure 4). In order to make priority to subsidized demand, a high unit cost is set to excess demand links associated to subsidized demand.

Furthermore, in case of total supply is greater than the total demand, Excess Supply Sub Network Representation is developed which consists of dummy retailer and the associated excess supply links (Figure 5). Dummy retailer is functioned to act as if receive product from any plants. This idea will guarantee the system to be always in balance condition.

Further problem is how to determine optimal solution of the NR, that is to determine the optimal flows and its associated paths of sending products from P_i' to R_i^S or R_i^C that cause minimum cost of the system. Actually this problem takes a form of Minimum Cost Flow (MCF) problem. Such problem is obviously analogous to the essence of mathematical problem of equation (1).

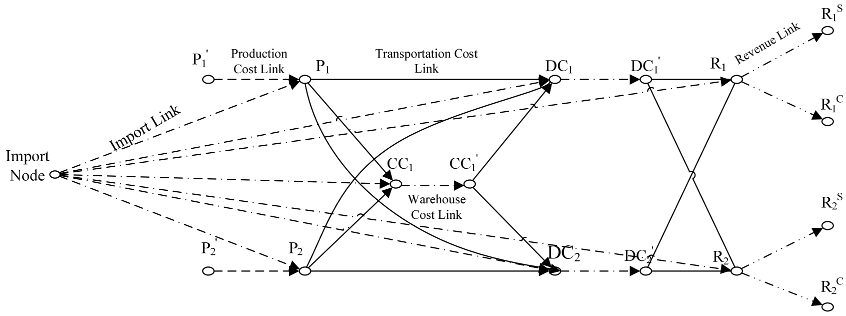

Figure 3 An Example of Import Sub Network Representation

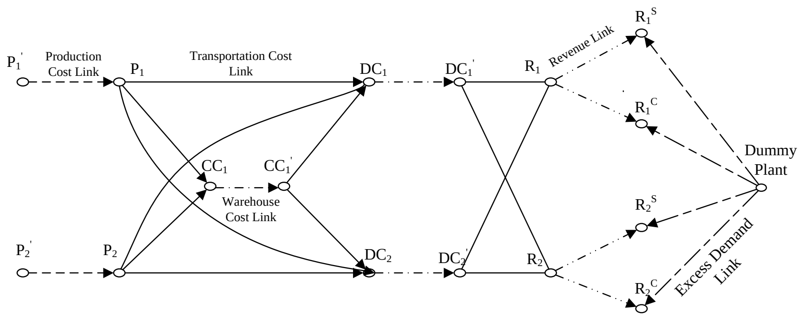

Figure 4 An Example of Demand Diminishing Sub Network Representation

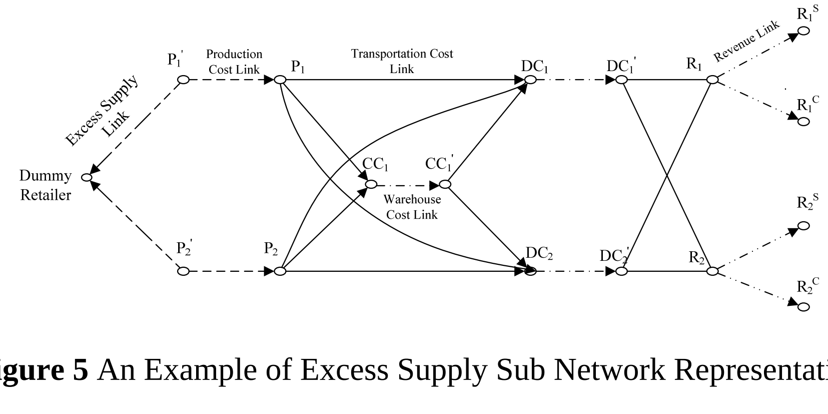

Figure 5 An Example of Excess Supply Sub Network Representation

FIXED CHARGE PROBLEM IN NETWORK REPRESENTATION

As described in equation (1), the location decision is represented mathematically by the use of binary variables X_c and Y_d which are valued as zero or one. Zero value denotes that the associated consolidation center or distribution center does not need to be opened, while one means the contrary. In NR, this variables do not exist and they are replaced by making use of heuristic solution. In equation (1), the cost related to binary variable is designated as fixed cost of facility, that is the cost incurred in establishing a warehouse or center. In NR, such cost is represented by fixed cost of facility links. Clearly, such links are not similar with other links in the context of their costing. Cost associated to transportation cost link, production cost link, as well as revenue link is a function of unit cost and the amount of flow passing through such link. While in fixed cost of facility link, the associated cost takes a form of fixed cost, in which it does not depend on the amount of flow. In network problem, such kind of costing is classified as fixed charge problem.

The idea of solving fixed charge facility location problem is coming from the fact that locating as many facilities as possible could reduce the transportation cost, while at the other side building more facilities means more cost (Daskin, 1995). The relation between both costs could be expressed in the form of total cost as shown in Figure 6. It can be seen that transportation cost decreases as the number of facilities increases, while the fixed cost linearly increases as the number of facilities increases. In this case, cost per facility is assumed to be similar for all facility. As a summation of both costs, total cost initially declines as the reduction in transportation cost that results from the addition of more facilities more than offsets the additional facility location costs. At some points, the cost of additional facilities exceeds the savings in transportation costs and the total cost increases as the facilities added.

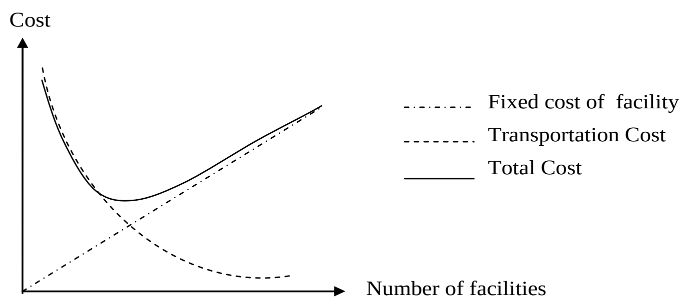

Figure 6 Relation Between Transportation, Facility, and Total Cost

A variety of heuristic algorithms have been devised for solving the fixed charge facility location problem. In case of uncapacitated problem, there exists DROP algorithm and ADD algorithm, as well as Lagrangian Relaxation approach and Dual-base approach (Daskin, 1995). Lagrangian Relaxation approach could be used also for capacitated problem.

Since this current study deals with the use of NR in solving location problem and the problem is characterized as capacitated problem, the heuristic approach based on the technique of ADD algorithm is introduced. The ADD algorithm is chosen since its nature is quite close to the character of NR, that is in the context of dominant visual components.

As noted in Figure 6, the total cost generally decreases as facilities are initially added to the solution. ADD algorithm greedily adds facilities to the solution until the algorithm fails to find a facility whose addition will result in a decrease in the total cost. Greedily means that each node that is added to the solution reduces the cost as much as possible, holding the previously selected sites fixed in the solution (Daskin, 1995). This mechanism is quite easy to be translated into NR, that is just by the principle of labelling the links associated to the fixed cost of facility. When certain warehouse or group of warehouses is being taken into consideration to be opened, links associated to such warehouses are just labeled with certain number, such as 1, and label 0 to other closed fixed cost of facility links.

In order to cope with the capacitated problem, the ADD algorithm in the sense of warehouse capacity is modified by the following mechanism. At the beginning, the capacity of all warehouses is set as unlimited. Everytime the best site of a warehouse is found and fixed in the solution, the capacity of such warehouse is set as its capacity. When the total capacity of all fixed warehouses is greater than either total supply (in the case of excess demand) or total demand (in the case of excess supply), all the warehouses are set as capacitated warehouses. This mechanism is expressed in the Figure 7.

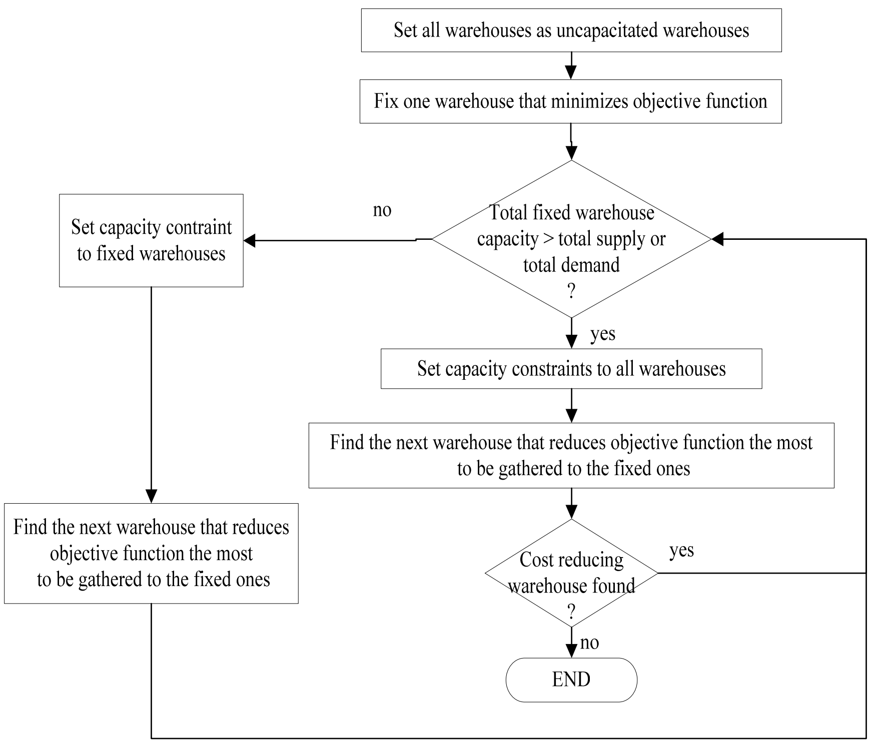

Figure 7 Step-Wise of Location Decision

The stepwise of determination of location decision is set as follows:

Step 1: It is assumed that all warehouses are uncapacitated ones. Fix one warehouse which minimizes the objective function the most, and set it as capacitated warehouse

Step 2: Check total capacity of all fixed warehouses. If it is lesser than the total demand or the total supply, go to step 3; otherwise, set all warehouses as capacitated ones and go to step 3.

Step 3: Find the next warehouse that reduces the objective function the most to be gathered to the current fixed ones. Primal-dual algorithm is applied to solve MCF problem of NR.

Step 4: If there is no more reduction in objective function, go to step 5; otherwise, go to step 2

Step 5: Set all fixed warehouses as the opened warehouses, and the remaining as the closed ones.

ILLUSTRATIVE EXAMPLE

In this section, an example of location decision problem of a simple distribution network consists of 2 plants and 2 retailers is discussed. Four cases are introduced to show the trend of values of objective function as the number of warehouses changes. Each case is differentiated by the type and number of potential warehouses, as well as the amount of demand and plant capacity (see Table 1).

Table 1 Type and Number of Potential Warehouses

Case	Type and number of potential warehouses
1	7 Consolidation Centers
2	4 Distribution Centers
3	4 Distribution Centers
4	7 Consolidation Centers

Figure 8 shows the value of objective funtion of each case. It is shown that at the initial, the objective function decreases as the number of facilities increases. As it reaches the minimum value, the objective function increases as the number of facilities increases. The optimal number of facilites is reached when the objective function at the minimum value.

Case 1 reaches the minimum value of the objective function if 3 warehouses are opened, while case 2 and case 4 come with 2 optimal warehouses. Case 3 shows slightly different trend of objective function. As the second warehouse opened, the value of objective function is drastically increases. Since in case 3 deals with capacitated warehouse, capacity of first opened warehouse is lesser than the total demand, so it is not sufficient to open just one warehouse. As the third warehouse opened, the objective function decreases and 3 warehouses becomes the optimal number of warehouses. Surely, step wise of Figure 7 guarantees that at the optimal condition total demand could always be accomodated by the total capacity of all opened warehouses.

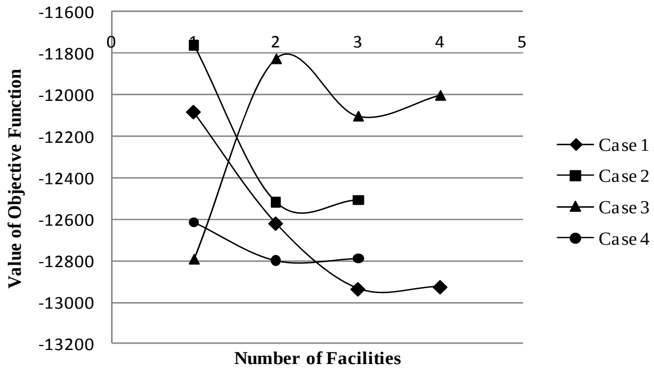

Figure 8 Trend of Value of Objective Function

CONCLUSION

This study proposes an approach of making location decision in distribution system of PSO-SOC which characterized by single commodity, multiplants, and capacitated warehouse problem. Basically the Network Representation was exploited to represent such model and modifiy ADD algorithm which originally deals with uncapacitated problem to make decision to open or close certain warehouse. The easiness and flexibility of the Network Representation in abstracting and comprehending the problem make NR useful for practitioner. The application of such approach should be extended to the bigger size of network to see its performance in solving various combinatoric network problems.

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