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Design of Vehicle Routes for Rice Distribution System in Bandung Using Simulated Annealing Algorithm

Peri Turnip[1], Nova Indah Saragih[2*]
[1)]Faculty of Science and Technology, Department of Mathematics, International Women University
Jl. Pasir Kaliki No.179 A, Bandung 40173
Email: peri.turnip@iwu.ac.id
[2)]School of Industrial Engineering, Department of Industrial Engineering, Telkom University
Jl. Telekomunikasi No. 1, Bandung 40257
Email: novaindah@telkomuniversity.ac.id

Abstract

VRP (vehicle routing problem) belongs to the problem of NP-hard. It makes the computation time becoming longer as the number of data increases. This statement is in accordance with previous research which solved the VRP for the rice distribution system in Bandung which consisted of 40 demand points in the form of traditional markets using the optimal method. The computational time required is 167 hours 52 minutes 38 seconds. Therefore, a heuristic method is needed to solve VRP with a more efficient computation time and an acceptable solution. The purpose of this paper is to complete the VRP by using an annealing simulation to design a rice distribution system in Bandung. Simulated annealing (SA) is a local search algorithm (meta-heuristic) that is able to get out of the local optimum. Its ease of implementation, the use of hill-climbing motions to avoid local optimal, and the convergent nature, have made SA a popular technique over the past two decades. After comparing the results to the ILP model, the computational experiments show what the SA algorithm developed in this paper is able to produce a relatively small gap in terms of total transportation cost, which is 1.26%. This paper has also succeeded in improving the previous paper by reducing the computation time to 19 seconds using the developed SA algorithm.

Keywords: *Heuristic method, rice distribution system, simulated annealing, vehicle routing problem*

Introduction

VRP (vehicle routing problem) is related to determining the optimal route set by the vehicle fleet to serve a certain set of customers. VRP is the most studied combinatorial optimization problem and one of the most important of them. The distribution of goods involves the service, within a certain period of time, from a group of customers by a group of vehicles, located at one or more depots, operated by a group of crews (drivers), and carrying out the movement using the appropriate road network. In particular, the VRP solution is linked to routes, each carried out by one vehicle starting and ending at their respective depots, so that the demand of all customers and all operational constraints are met, and total transportation costs are minimized.

The road network used for the transportation of goods is generally depicted with a graph, the arcs of which represent the road segments and their nodes according to road junctions and the location of depots and customers. An arc may or may not be directional, depending on whether only the arc can be traversed in one direction (e.g., due to a one-way street, typical of urban networks or toll roads) or both directions. Each arc has a cost, which generally represents its length, and a travel time, which may depend on the type of vehicle or on the period in which the arc is traversed (Toth & Vigo, 2002).

VRP belongs to the problem of NP-hard. It makes the computation time becomes longer as the number of data increases. This statement is in accordance with research conducted by (Saragih & Rachman, 2020) which solved the VRP for the rice distribution system in Bandung

City. The rice distribution system consists of 40 demand points in the form of traditional markets. The computational time required is 167 hours 52 minutes 38 seconds. Therefore, a heuristic method is needed to solve VRP with a more efficient computation time and an acceptable solution.

The name SA is given because of its analogy with the metal quenching process, in which solid crystals are heated and then very slowly cooled until they reach the most ordered configuration of the crystal lattice (its minimum energy lattice state), and thus free from crystals. disabled. If the schedule of cooling is slow enough, the final configuration will result in a density with superior structural integrity. SA develops the relationship between this type of thermodynamic behavior and the search for global minimum on discrete optimization problems. Furthermore, SA provides an algorithmic way to exploit this relationships (Henderson et al., 2003).

In each iteration of SA applied to a discrete optimization problem, the objective function returns values for two solutions, namely the current solution and the selected new solution which are then compared. The improved solution is always accepted, while a small part of the solution that is not improved (inferior) is accepted in the hope of getting out of the local optimum in search of the global optimum. The probability of receiving a solution is not fixed depending on the temperature parameter, which usually does not increase with the iteration of the algorithm (Henderson et al., 2003).

The main algorithmic feature of SA is that it provides a means of exiting the local optimum by enabling hill-climbing movements (movements that worsen the value of the objective function). When the temperature parameter decreases to zero, hill-climbing motions occur less frequently, and the distribution of solutions associated with the non-homogeneous Markov chain modeling the algorithm's behavior converges to a form in which all probabilities are concentrated at the global optimum. the set of solutions (provided the algorithm converges, otherwise the algorithm will converge to a local optimum, which may or may not be a global optimum) (Henderson et al., 2003).

There were number of studies that have solved VRP. (Lubis et al., 2016) solved VRP using the nearest neighbor algorithm. (Lubis et al., 2016) applied the VRP to waste transportation in urban areas. (Fitria et al., 2009) solved VRP using a sequential insertion algorithm and applied it to garbage collection and transportation trucks in Bandung. (Wulandari et al., 2019) used a heuristic method to solve the VRP, namely the Palgunadi algorithm. (Ramadhan & Imran, 2018) solved VRP using a heuristic method, namely the record-to-record travel algorithm. (Ramadhan & Imran, 2018) applied the VRP to the case of school buses in the city of Bandung. (Saragih & Rachman, 2020) solved VRP by formulating the problem as an ILP (integer linear programming) model and applied it to the rice distribution system in Bandung City. (Sopandi et al., 2020) solved VRP improved nearest neighbor heuristic to continue (Saragih & Rachman, 2020).

The purpose of this paper is to complete the VRP by using annealing simulation to design a system distribution of rice in Bandung. The heuristic method used to solve VRP in this paper is annealing simulation or SA. SA is a local search algorithm (meta-heuristic) that is able to get out of the local optimum. Its ease of implementation, the use of hill-climbing motions to avoid local optimal, and the convergent nature, have made SA a popular technique over the past two decades. Discrete optimization problems are usually solved by SA (Henderson et al., 2003).

Methods

Several terms are needed to define the specific features of SA for discrete optimization problems. They are:

Ω set of all possible solutions
$f(\omega*)$ global minimum value
$N(\omega)$ the neighborhood function
T temperature
t_0 initial temperature
t_k temperature drop schedule
M_k iteration schedule
P probability of acceptance

For example, Ω is the set of all possible solutions. The objective function is defined as $f:\Omega\rightarrow\Re$. The goal to be achieved is to produce a global minimum, $\omega*$ ($\omega*\in\Omega$ so that $f(\omega) \geq f(\omega*)$ for all $\omega\in\Omega$. The objective function must be constrained to ensure that $\omega*$ exists. Define $N(\omega)$ as the function of neighborhood for $\omega\in\Omega$.

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Therefore, associated with each solution, $\omega \in \Omega$ is the neighboring solution, $N(\omega)$, which can be reached in one iteration of the local search algorithm.

SA starts with an initial solution $\omega \in \Omega$. Then neighboring solutions are generated $\omega' \in N(\omega)$ randomly or using certain rules. The solution of candidate, ω', is accepted as the new solution based on the following opportunity of acceptance:

$$P \begin{cases} \exp -[f(\omega') - f(\omega)/t_k] \; if \; f(\omega') - f(\omega) > 0 \\ 1 \; if \; f(\omega') - f(\omega) \leq 0 \end{cases}$$ Eq. 1

Define as the parameter of temperature in the *k*-th (outer loop) iteration so that:

$$t_k \geq 0 \; for \; all \; k \; and \; \lim_{k \to +\infty} t_k = 0$$ Eq. 2

This opportunity of acceptance is a basic element in SA for the search mechanism. If the temperature *T* is reduced slowly enough, the system can reach a steady state in *k* iterations.

The SA pseudo-code is given as follows:

Decide the initial solution $\omega \in \Omega$
Decide change of temperature counter *k* = 0
Decide a drop of temperature schedule, t_k
Decide the temperature of initial $T = t_0 \geq 0$
Decide the schedule of iteration, M_k, which describes the number of iterations executed at each temperature t_k
Redo
Set the counter repeat *m* = 0
Redo
Produce a solution $\omega' \in N(\omega)$
Compute $\Delta_{\omega,\omega'} = f(\omega') - f(\omega)$
If $\Delta_{\omega,\omega'} \leq 0$, then $\omega \leftarrow \omega'$
If $\Delta_{\omega,\omega'} > 0$, then $\omega \leftarrow \omega'$ with probability $\exp -[\Delta_{\omega,\omega'}/t_k]$
$m \leftarrow m + 1$
Until $m = M_k$
$k \leftarrow k + 1$
Until the termination criteria are met.

The SA formulation yields $M_0 + M_1 + \cdots + M_k$ total iterations executed where k corresponds to the value for t_k where the termination criteria are met. In addition, if $M_k = 1$ for all *k*, then the temperature changes with each iteration.

The SA algorithm developed in this paper is from (Saragih et al., 2019) that can be seen in Fig. 1. The efficiency of the algorithm will be evaluated by comparing the results with the ILP (integer linear programming) model from (Saragih & Rachman, 2020). The ILP model is given as follows.

Sets
V vehicles set
K markets set
M merged set of central market of Caringin and markets

Indices
v vehicles index
k markets index

Parameters
d_{kl} cost of transportation between node k dan node l (Rp) $(\forall k, l \in M)$
vc capacity of vehicle (sack)
μ_k demand of markets k (sack) $(k \in K)$
B amount of markets consisted in set K , i.e $B = |K|$

Decision variabels
$R_{klv} \begin{cases} 1 \text{ if } k \text{ precedes } l \text{ in route of vehicle } v \\ 0 \text{ otherwise} \end{cases}$ $(\forall k, l \in M, \forall v \in V)$
M_{kv} variable of auxiliary defined for markets k for subtour elimination in vehicle route v $(\forall k \in K, \forall v \in V)$
OT total cost of transportation (Rp)

The model
$$\text{Minimize } OT = \sum_{v \in V} \sum_{k \in M} \sum_{l \in M} d_{kl} R_{klv}$$ Eq. 3

Subject to
$$\sum_{v \in V} \sum_{l \in M} R_{klv} = 1, \forall k \in K$$ Eq. 4
$$\sum_{l \in K} \sum_{k \in M} \mu_l R_{klv} \leq vc, \forall v \in V$$ Eq. 5
$$M_{kv} - M_{lv} + (B \times R_{klv}) \leq B - 1, \forall k, l \in K, \forall v \in V$$ Eq. 6
$$\sum_{l \in M} R_{klv} - \sum_{l \in M} R_{lkv} = 0, \forall k \in M, \forall v \in V$$ Eq. 7
$$R_{klv} \in \{0,1\}, \forall k, l \in M, \forall v \in V$$ Eq. 8
$$M_{kv} \geq 0, \forall k \in K, \forall v \in V$$ Eq. 9

Eq. 3 is the objective function which is the total cost of transportation. Eq. 4 guarantees that every market is served by a vehicle exactly once. Eq. 5 guarantees that the goods sent by the vehicle do not exceed its capacity. Eq. 6 is the subtour elimination constraint. Eq. 7 is the flow conservation constraint. Eq. 8-9 is the decision variable constraint.

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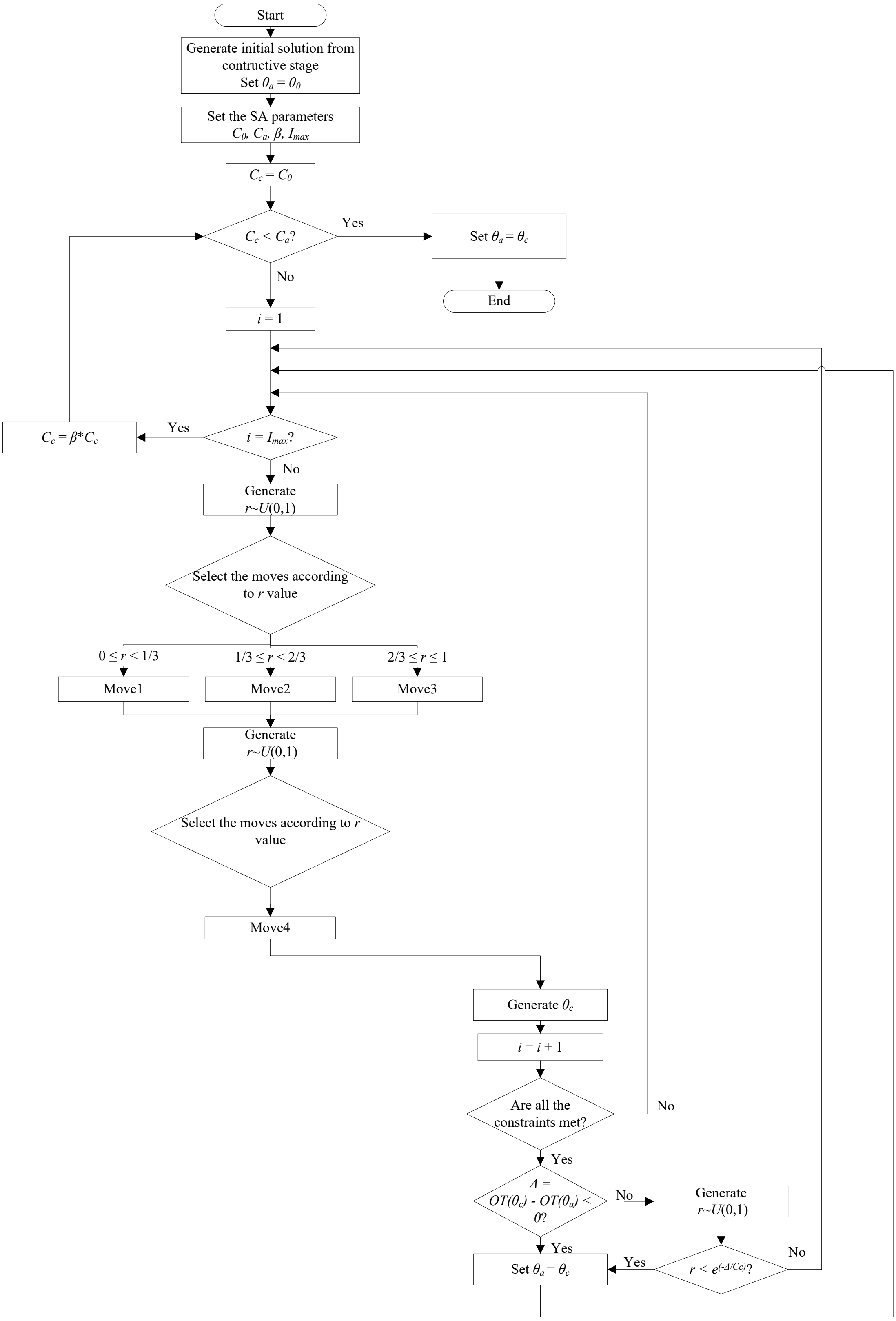

Figure 1. The SA algorithm

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Results and Discussion

The Data

Performance of the SA algorithm is evaluated using several instances. Table 1 and Table 2 give data used for the evaluation. Capacity of the vehicle is 50. Table 1 is the data of the demand for the nodes.

Table 1. Demand of the nodes

Node	Demand
1	0
2	20
3	15
4	20
5	10
6	30
7	15
8	25
9	10
10	30
11	20

Table 2 is the data of the distance for the nodes.

Table 2. Distance of the nodes

Nodes										
1	2	3	4	5	6	7	8	9	10	11
0	3	2	1	5	4	2	3	1	1	3
3	0	4	2	3	1	5	1	3	3	4
2	4	0	5	2	1	3	4	5	3	2
1	2	5	0	2	1	4	2	4	1	5
5	3	2	2	0	4	1	5	2	4	1
4	1	1	1	4	0	5	2	4	5	4
2	5	3	4	1	5	0	4	1	1	3
3	1	4	2	5	2	4	0	5	4	2
1	3	5	4	2	4	1	5	0	2	5
1	3	3	1	4	5	1	4	2	0	3
3	4	2	5	1	4	3	2	5	3	0

The Comparison to the Previous Study

To obtain the performance of the SA algorithm developed, comparison between solution resulted from the SA algorithm and the ILP model is performed. The comparison is given in Table 3 and Table 4. The average gap between both of the solutions is relatively small which is 1.26%.

Table 3. The comparison between solutions (1)

No.	#Traditional markets	SA algorithm	
		Total cost of transportation (Rp)	Time of CPU (s)
1	2	9	2
2	3	9	2
3	4	15	3
4	5	16	3
5	6	19	3
6	7	20	3
7	8	20	4
8	9	23	4

Table 4. The comparison between solutions (2)

No.	ILP model		Gap
	Total cost of transportation (Rp)	Time of CPU (s)	Total cost of transportation (%)
1	9	0	0.00
2	9	0	0.00
3	15	2	0.00
4	16	12	0.00
5	18	118	5.56
6	20	657	0.00
7	20	2232	0.00
8	22	9598	4.55
		Average	1.26

The Determination of the Routes

Since the gap between the SA algorithm and the ILP model is relatively small, it can be said that the SA algorithm developed is able to produce an acceptable solution that can be used for the real system which is the rice distribution system in Bandung. Data of demand are from (Saragih & Rachman, 2020) and data of transportation cost are given in the appendix. Design of vehicle routes for the rice distribution system in Bandung is given in Table 5.

There are 38 routes needed to serve 40 traditional markets in Bandung. Total transportation cost resulted is Rp39.024.269. Computational time needed is 19 seconds.

Conclusion

This paper has successfully solved the VRP using the SA algorithm in the distribution system of rice in Bandung. After comparing the results to the ILP model, the computational experiments shows that the SA algorithm developed in this paper is able to produce a relatively small gap in terms of total transportation cost, which is 1.26%.

Table 5. Vehicle routes resulted

No.	Name of the routes	Vehicle routes
1	Route1	[1,13,1]
2	Route2	[1,12,1]
3	Route3	[1,4,1]
4	Route4	[1,5,1]
5	Route5	[1,2,1]
6	Route6	[1,3,1]
7	Route7	[1,20,1]
8	Route8	[1,19,1]
9	Route9	[1,35,1]
10	Route10	[1,34,1]
11	Route11	[1,36,1]
12	Route12	[1,37,1]
13	Route13	[1,38,1]
14	Route14	[1,15,1]
15	Route15	[1,14,1]
16	Route16	[1,24,1]
17	Route17	[1,23,1]
18	Route18	[1,22,1]
19	Route19	[1,21,1]
20	Route20	[1,9,10,18,1]
21	Route21	[1,17,1]
22	Route22	[1,16,1]
23	Route23	[1,25,1]
24	Route24	[1,27,1]
25	Route25	[1,26,1]
26	Route26	[1,39,1]
27	Route27	[1,28,1]
28	Route28	[1,29,1]
29	Route29	[1,32,1]
30	Route30	[1,33,1]
31	Route31	[1,11,1]
32	Route32	[1,8,1]
33	Route33	[1,7,1]
34	Route34	[1,6,1]
35	Route35	[1,31,1]
36	Route36	[1,30,1]
37	Route37	[1,41,1]
38	Route38	[1,40,1]

This paper also has successfully improved the previous paper by reducing the computational time to 19 seconds using the SA algorithm. Developing other methods to solve the VRP, such as Tabu Search and Genetic Algorithm, can be interesting topics for future works.

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The Appendix. Transportation costs (Rp)

Node	1	2	3	4	5	6	7	8	9	10
1	0	23.476	21.52	20.542	21.031	18.585	18.585	18.585	40.105	41.573
2	23.476	0	3.913	21.031	5.869	21.52	21.52	21.52	34.236	30.324
3	21.52	3.913	0	18.585	22.987	23.965	23.965	23.965	24.944	23.965
4	20.542	21.031	18.585	0	5.869	27.878	27.878	27.878	24.944	27.389
5	21.031	5.869	22.987	5.869	0	24.455	24.455	24.455	25.433	22.987
6	18.585	21.52	23.965	27.878	24.455	0	0	0	43.04	42.551
7	18.585	21.52	23.965	27.878	24.455	0	0	0	43.04	42.551
8	18.585	21.52	23.965	27.878	24.455	0	0	0	43.04	42.551
9	40.105	34.236	24.944	24.944	25.433	43.04	43.04	43.04	0	6.847
10	41.573	30.324	23.965	27.389	22.987	42.551	42.551	42.551	6.847	0
11	15.651	9.782	22.987	12.716	9.782	20.053	20.053	20.053	37.66	36.682
12	13.695	16.629	23.476	7.825	13.205	27.389	27.389	27.389	29.345	31.302
13	13.695	16.629	23.476	7.825	13.205	27.389	27.389	27.389	29.345	31.302
14	47.931	33.747	27.389	31.302	28.856	46.464	46.464	46.464	8.804	11.738
15	47.931	33.747	27.389	31.302	28.856	46.464	46.464	46.464	8.804	11.738
16	44.996	43.529	36.193	33.258	30.813	55.756	55.756	55.756	9.293	16.14
17	44.996	43.529	36.193	33.258	30.813	55.756	55.756	55.756	9.293	16.14
18	44.996	43.529	36.193	33.258	30.813	55.756	55.756	55.756	9.293	16.14
19	31.302	17.118	10.271	23.476	15.651	29.345	29.345	29.345	27.389	24.944
20	24.944	4.891	3.913	17.607	14.184	24.455	24.455	24.455	28.856	26.411
21	52.333	40.105	33.258	38.638	34.725	52.333	52.333	52.333	15.651	18.585
22	52.333	40.105	33.258	38.638	34.725	52.333	52.333	52.333	15.651	18.585
23	46.464	34.236	27.389	33.258	28.856	46.464	46.464	46.464	10.271	13.205
24	46.464	34.236	27.389	33.258	28.856	46.464	46.464	46.464	10.271	13.205
25	44.018	41.084	39.127	30.813	31.302	54.289	54.289	54.289	15.162	21.52
26	53.8	51.354	48.909	43.529	44.996	64.56	64.56	64.56	25.433	31.791
27	54.289	53.8	47.442	41.084	45.974	66.027	66.027	66.027	24.944	28.367
28	29.345	27.389	28.367	17.607	18.096	41.084	41.084	41.084	17.607	23.965
29	29.345	27.389	28.367	17.607	18.096	41.084	41.084	41.084	17.607	23.965
30	49.398	63.582	64.56	50.865	54.289	68.473	68.473	68.473	48.42	56.245
31	49.398	63.582	64.56	50.865	54.289	68.473	68.473	68.473	48.42	56.245
32	22.987	24.944	25.433	13.695	14.184	37.171	37.171	37.171	20.053	21.52
33	27.878	14.673	15.651	12.716	14.184	34.725	34.725	34.725	18.096	19.075
34	41.084	19.075	19.075	22.987	20.542	41.084	41.084	41.084	24.455	21.52
35	41.084	19.075	19.075	22.987	20.542	41.084	41.084	41.084	24.455	21.52
36	41.084	19.075	19.075	22.987	20.542	41.084	41.084	41.084	24.455	21.52
37	55.267	32.28	31.791	35.704	33.258	56.245	56.245	56.245	37.66	35.214
38	55.267	32.28	32.28	39.127	36.682	45.974	45.974	45.974	42.551	39.616
39	31.302	31.791	29.345	18.096	18.585	41.573	41.573	41.573	10.76	17.607
40	78.743	72.385	72.385	80.7	66.516	90.971	90.971	90.971	49.887	53.8
41	78.743	72.385	72.385	80.7	66.516	90.971	90.971	90.971	49.887	53.8

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Node	11	12	13	14	15	16	17	18	19	20
1	15.651	13.695	13.695	47.931	47.931	44.996	44.996	44.996	31.302	24.944
2	9.782	16.629	16.629	33.747	33.747	43.529	43.529	43.529	17.118	4.891
3	22.987	23.476	23.476	27.389	27.389	36.193	36.193	36.193	10.271	3.913
4	12.716	7.825	7.825	31.302	31.302	33.258	33.258	33.258	23.476	17.607
5	9.782	13.205	13.205	28.856	28.856	30.813	30.813	30.813	15.651	14.184
6	20.053	27.389	27.389	46.464	46.464	55.756	55.756	55.756	29.345	24.455
7	20.053	27.389	27.389	46.464	46.464	55.756	55.756	55.756	29.345	24.455
8	20.053	27.389	27.389	46.464	46.464	55.756	55.756	55.756	29.345	24.455
9	37.66	29.345	29.345	8.804	8.804	9.293	9.293	9.293	27.389	28.856
10	36.682	31.302	31.302	11.738	11.738	16.14	16.14	16.14	24.944	26.411
11	0	9.782	9.782	36.193	36.193	36.682	36.682	36.682	18.585	12.227
12	9.782	0	0	35.214	35.214	33.747	33.747	33.747	29.345	22.987
13	9.782	0	0	35.214	35.214	33.747	33.747	33.747	29.345	22.987
14	36.193	35.214	35.214	0	0	9.782	9.782	9.782	22.987	24.455
15	36.193	35.214	35.214	0	0	9.782	9.782	9.782	22.987	24.455
16	36.682	33.747	33.747	9.782	9.782	0	0	0	34.236	35.704
17	36.682	33.747	33.747	9.782	9.782	0	0	0	34.236	35.704
18	36.682	33.747	33.747	9.782	9.782	0	0	0	34.236	35.704
19	18.585	29.345	29.345	22.987	22.987	34.236	34.236	34.236	0	8.315
20	12.227	22.987	22.987	24.455	24.455	35.704	35.704	35.704	8.315	0
21	42.062	42.062	42.062	6.847	6.847	15.651	15.651	15.651	23.476	30.813
22	42.062	42.062	42.062	6.847	6.847	15.651	15.651	15.651	23.476	30.813
23	36.193	36.682	36.682	1.712	1.712	14.184	14.184	14.184	17.607	24.944
24	36.193	36.682	36.682	1.712	1.712	14.184	14.184	14.184	17.607	24.944
25	37.171	34.236	34.236	15.162	15.162	5.869	5.869	5.869	32.28	39.127
26	49.887	45.974	45.974	25.433	25.433	16.14	16.14	16.14	42.062	49.398
27	47.442	44.996	44.996	20.053	20.053	16.629	16.629	16.629	38.149	44.507
28	23.965	20.053	20.053	23.476	23.476	20.053	20.053	20.053	27.878	32.769
29	23.965	20.053	20.053	23.476	23.476	20.053	20.053	20.053	27.878	32.769
30	53.8	45.974	45.974	53.8	53.8	46.464	46.464	46.464	61.625	66.516
31	53.8	45.974	45.974	53.8	53.8	46.464	46.464	46.464	61.625	66.516
32	20.053	13.205	13.205	26.411	26.411	24.455	24.455	24.455	25.433	30.324
33	19.075	20.542	20.542	24.455	24.455	25.433	25.433	25.433	20.542	27.389
34	26.9	35.214	35.214	19.564	19.564	30.813	30.813	30.813	8.804	16.14
35	26.9	35.214	35.214	19.564	19.564	30.813	30.813	30.813	8.804	16.14
36	26.9	35.214	35.214	19.564	19.564	30.813	30.813	30.813	8.804	16.14
37	40.105	44.996	44.996	33.258	33.258	44.507	44.507	44.507	22.009	29.345
38	40.594	101.24	101.24	37.66	37.66	48.909	48.909	48.909	25.433	29.345
39	24.455	22.009	22.009	16.14	16.14	13.695	13.695	13.695	21.52	26.411
40	72.385	69.94	69.94	44.996	44.996	41.573	41.573	41.573	62.604	69.451
41	72.385	69.94	69.94	44.996	44.996	41.573	41.573	41.573	62.604	69.451

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Node	21	22	23	24	25	26	27	28	29	30
1	52.333	52.333	46.464	46.464	44.018	53.8	54.289	29.345	29.345	49.398
2	40.105	40.105	34.236	34.236	41.084	51.354	53.8	27.389	27.389	63.582
3	33.258	33.258	27.389	27.389	39.127	48.909	47.442	28.367	28.367	64.56
4	38.638	38.638	33.258	33.258	30.813	43.529	41.084	17.607	17.607	50.865
5	34.725	34.725	28.856	28.856	31.302	44.996	45.974	18.096	18.096	54.289
6	52.333	52.333	46.464	46.464	54.289	64.56	66.027	41.084	41.084	68.473
7	52.333	52.333	46.464	46.464	54.289	64.56	66.027	41.084	41.084	68.473
8	52.333	52.333	46.464	46.464	54.289	64.56	66.027	41.084	41.084	68.473
9	15.651	15.651	10.271	10.271	15.162	25.433	24.944	17.607	17.607	48.42
10	18.585	18.585	13.205	13.205	21.52	31.791	28.367	23.965	23.965	56.245
11	42.062	42.062	36.193	36.193	37.171	49.887	47.442	23.965	23.965	53.8
12	42.062	42.062	36.682	36.682	34.236	45.974	44.996	20.053	20.053	45.974
13	42.062	42.062	36.682	36.682	34.236	45.974	44.996	20.053	20.053	45.974
14	6.847	6.847	1.712	1.712	15.162	25.433	20.053	23.476	23.476	53.8
15	6.847	6.847	1.712	1.712	15.162	25.433	20.053	23.476	23.476	53.8
16	15.651	15.651	14.184	14.184	5.869	16.14	16.629	20.053	20.053	46.464
17	15.651	15.651	14.184	14.184	5.869	16.14	16.629	20.053	20.053	46.464
18	15.651	15.651	14.184	14.184	5.869	16.14	16.629	20.053	20.053	46.464
19	23.476	23.476	17.607	17.607	32.28	42.062	38.149	27.878	27.878	61.625
20	30.813	30.813	24.944	24.944	39.127	49.398	44.507	32.769	32.769	66.516
21	0	0	4.891	4.891	24.455	32.28	22.498	32.28	32.28	62.604
22	0	0	4.891	4.891	24.455	32.28	22.498	32.28	32.28	62.604
23	4.891	4.891	0	0	18.096	27.878	22.498	25.922	25.922	57.224
24	4.891	4.891	0	0	18.096	27.878	22.498	25.922	25.922	57.224
25	24.455	24.455	18.096	18.096	0	12.227	10.76	23.965	23.965	42.062
26	32.28	32.28	27.878	27.878	12.227	0	21.52	26.411	26.411	30.813
27	22.498	22.498	22.498	22.498	10.76	21.52	0	30.813	30.813	48.909
28	32.28	32.28	25.922	25.922	23.965	26.411	30.813	0	0	34.236
29	32.28	32.28	25.922	25.922	23.965	26.411	30.813	0	0	34.236
30	62.604	62.604	57.224	57.224	42.062	30.813	48.909	34.236	34.236	0
31	62.604	62.604	57.224	57.224	42.062	30.813	48.909	34.236	34.236	0
32	36.193	36.193	26.9	26.9	28.367	29.834	35.214	7.825	7.825	37.171
33	35.214	35.214	22.987	22.987	28.856	32.28	35.704	16.14	16.14	50.376
34	23.476	23.476	19.564	19.564	35.704	44.018	38.149	38.149	38.149	71.896
35	23.476	23.476	19.564	19.564	35.704	44.018	38.149	38.149	38.149	71.896
36	23.476	23.476	19.564	19.564	35.704	44.018	38.149	38.149	38.149	71.896
37	34.725	34.725	33.258	33.258	49.398	59.18	51.844	51.844	51.844	88.036
38	44.507	44.507	37.66	37.66	53.8	62.114	56.245	57.713	57.713	91.46
39	24.944	24.944	18.585	18.585	17.607	20.542	24.455	9.782	9.782	44.018
40	47.442	47.442	47.442	47.442	35.704	47.931	28.367	63.582	63.582	49.887
41	47.442	47.442	47.442	47.442	35.704	47.931	28.367	63.582	63.582	49.887

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Node	31	32	33	34	35	36	37	38	39	40	41
1	49.398	22.987	27.878	41.084	41.084	41.084	55.267	55.267	31.302	78.743	78.743
2	63.582	24.944	14.673	19.075	19.075	19.075	32.28	32.28	31.791	72.385	72.385
3	64.56	25.433	15.651	19.075	19.075	19.075	31.791	32.28	29.345	72.385	72.385
4	50.865	13.695	12.716	22.987	22.987	22.987	35.704	39.127	18.096	80.7	80.7
5	54.289	14.184	14.184	20.542	20.542	20.542	33.258	36.682	18.585	66.516	66.516
6	68.473	37.171	34.725	41.084	41.084	41.084	56.245	45.974	41.573	90.971	90.971
7	68.473	37.171	34.725	41.084	41.084	41.084	56.245	45.974	41.573	90.971	90.971
8	68.473	37.171	34.725	41.084	41.084	41.084	56.245	45.974	10.76	90.971	90.971
9	48.42	20.053	18.096	24.455	24.455	24.455	37.66	42.551	10.76	49.887	49.887
10	56.245	21.52	19.075	21.52	21.52	21.52	35.214	39.616	17.607	53.8	53.8
11	53.8	20.053	19.075	26.9	26.9	26.9	40.105	40.594	24.455	72.385	72.385
12	45.974	13.205	20.542	35.214	35.214	35.214	44.996	101.24	22.009	69.94	69.94
13	45.974	13.205	20.542	35.214	35.214	35.214	44.996	101.24	22.009	69.94	69.94
14	53.8	26.411	24.455	19.564	19.564	19.564	33.258	37.66	16.14	44.996	44.996
15	53.8	26.411	24.455	19.564	19.564	19.564	33.258	37.66	16.14	44.996	44.996
16	46.464	24.455	25.433	30.813	30.813	30.813	44.507	48.909	13.695	41.573	41.573
17	46.464	24.455	25.433	30.813	30.813	30.813	44.507	48.909	13.695	41.573	41.573
18	46.464	24.455	25.433	30.813	30.813	30.813	44.507	48.909	13.695	41.573	41.573
19	61.625	25.433	20.542	8.804	8.804	8.804	22.009	25.433	21.52	62.604	62.604
20	66.516	30.324	27.389	16.14	16.14	16.14	29.345	29.345	26.411	69.451	69.451
21	62.604	36.193	35.214	23.476	23.476	23.476	34.725	44.507	24.944	47.442	47.442
22	62.604	36.193	35.214	23.476	23.476	23.476	34.725	44.507	24.944	47.442	47.442
23	57.224	26.9	22.987	19.564	19.564	19.564	33.258	37.66	18.585	47.442	47.442
24	57.224	26.9	22.987	19.564	19.564	19.564	33.258	37.66	18.585	47.442	47.442
25	42.062	28.367	28.856	35.704	35.704	35.704	49.398	53.8	17.607	35.704	35.704
26	30.813	29.834	32.28	44.018	44.018	44.018	59.18	62.114	20.542	47.931	47.931
27	48.909	35.214	35.704	38.149	38.149	38.149	51.844	56.245	24.455	28.367	28.367
28	34.236	7.825	16.14	38.149	38.149	38.149	51.844	57.713	9.782	63.582	63.582
29	34.236	7.825	16.14	38.149	38.149	38.149	51.844	57.713	9.782	63.582	63.582
30	0	37.171	50.376	71.896	71.896	71.896	88.036	91.46	44.018	49.887	49.887
31	0	37.171	50.376	71.896	71.896	71.896	88.036	91.46	44.018	49.887	49.887
32	37.171	0	13.695	36.193	36.193	36.193	49.887	56.245	9.782	68.962	68.962
33	50.376	13.695	0	23.965	23.965	23.965	39.127	40.594	10.76	58.691	58.691
34	71.896	36.193	23.965	0	0	0	15.651	25.433	26.9	67.984	67.984
35	71.896	36.193	23.965	0	0	0	15.651	25.433	26.9	67.984	67.984
36	71.896	36.193	23.965	0	0	0	15.651	25.433	26.9	67.984	67.984
37	88.036	49.887	39.127	15.651	15.651	15.651	0	13.695	40.594	77.276	77.276
38	91.46	56.245	40.594	25.433	25.433	25.433	13.695	0	44.507	82.656	82.656
39	44.018	9.782	10.76	26.9	26.9	26.9	40.594	44.507	0	47.931	47.931
40	49.887	68.962	58.691	67.984	67.984	67.984	77.276	82.656	47.931	0	0
41	49.887	68.962	58.691	67.984	67.984	67.984	77.276	82.656	47.931	0	0