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Introducing Copula Functions to Estimate the Reliability of Dependent Mechanical Systems

Adhitya Ryan Ramadhani[1], Waskito Pranowo[2]
[1)] Faculty of Industrial Technology, Department of Mechanical Engineering. Universitas Pertamina
Jl. Teuku Nyak Arief, Jakarta Selatan 12220
Email: adhitya.rr@universitaspertamina.ac.id
[2)] Faculty of Exploration and Production Technology, Department of Geophysical Engineering. Universitas Pertamina
Jl. Teuku Nyak Arief, Jakarta Selatan 12220
Email: waskito.pranowo@universitaspertamina.ac.id

Abstract

This paper addresses the challenge of assessing the reliability of complex mechanical systems where components are inherently correlated in their failure modes. Traditionally, the assumption of independence among these components has been employed, but it often fails to capture the real-world complexities. To overcome this limitation, copula functions are introduced as a robust methodology for modeling the dependent relationships between correlated variables within mechanical systems. This paper aims to demonstrate the utility of copulas in estimating system reliability while accounting for these dependencies. The results reveal that the Clayton copula emerges as the most suitable model for representing dependence in such systems. Importantly, the reliability estimates obtained through copula-based methods not only reflect the complex interdependencies accurately but also align with the principles of the boundary theory of reliability. This research underscores the potential of copula-based reliability estimation as a valuable alternative, offering a more comprehensive and precise assessment of reliability in complex mechanical systems and holding significant promise for practical engineering applications. This framework allows the consideration of dependence among the observed variables that is usually overlooked in engineering practice.

Keywords: *Reliability, Copula functions, Mechanical system, Dependent case, Correlation*

Abstrak

Paper ini mengatasi tantangan dalam menilai keandalan sistem mekanikal kompleks di mana komponen-komponennya secara inheren berkorelasi dalam mode kegagalan mereka. Secara tradisional, asumsi independen di antara komponen-komponen ini telah digunakan, tetapi seringkali gagal untuk menangkap kompleksitas dunia nyata. Untuk mengatasi keterbatasan ini, fungsi copula diperkenalkan sebagai metodologi yang kuat untuk memodelkan hubungan yang saling bergantung antara variabel-variabel yang berkorelasi dalam sistem mekanikal. Paper ini bertujuan untuk mendemonstrasikan kegunaan copula dalam memperkirakan keandalan sistem dengan mempertimbangkan dependensi ini. Hasil penelitian menunjukkan bahwa copula Clayton muncul sebagai model yang paling cocok untuk merepresentasikan dependensi dalam sistem-sistem seperti itu. Yang lebih penting, estimasi keandalan yang diperoleh melalui metode berbasis copula tidak hanya mencerminkan interdependensi yang kompleks dengan akurat, tetapi juga sejalan dengan prinsip-prinsip teori batas keandalan. Penelitian ini menggarisbawahi potensi estimasi keandalan berbasis copula sebagai alternatif yang aplikatif, menawarkan penilaian keandalan yang lebih komprehensif dan akurat dalam sistem mekanikal kompleks dan memiliki potensi besar untuk aplikasi rekayasa praktis. Metode yang diusulkan dapat digunakan untuk memodelkan hubungan antar variabel yang sering kali diabaikan dalam praktik rekayasa.

Kata kunci: Keandalan, Fungsi copula, Sistem mekanikal, Kasus dependensi, Korelasi

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Introduction

Reliability assessment for mechanical systems is essential to keep all components functioning. There has been various research on how reliability assessment is approached considering the lifetime of each mechanical component. Traditional reliability approaches were used to optimize maintenance strategy, estimate the reliability level of a machine, and assign maintenance tasks based on failure modes (Aritonang, 2023; Maukar et al., 2016; Mustaqim et al., 2020). Li et al. (2023) proposed a model incorporating machine production rate to analyze preventive maintenance activities. They used multiple degradation-driven preventive maintenance to achieve a more comprehensive solution. Louzada et al. (2022) used Monte Carlo (MC) simulation to evaluate failure times under a manual repair framework. In addition, Avontuur & van der Werff (2002) analyzed the reliability of mechanical and hydraulic systems based on finite element analysis. The model was able to describe the non-linear behavior of the systems. The reliability of mechanical systems was improved by introducing other advanced techniques such as the Wiener process, inverse Gauss, and accelerated life testing (ALT) to model the failure mechanisms (de Reffye & Dersin, 2010; Jiang et al., 2023; Woo & O'Neal, 2019). Non-homogeneous Poisson process (NHPP) was also used to optimize the reliability of systems under some stresses (Feizabadi & Jahromi, 2017; Wei & Liu, 2023). The proposed models were able to reduce procurement and maintenance costs. However, computational issues are always found when assessing reliability using traditional approaches (Hu et al., 2019). Furthermore, mechanical systems are more challenging when dealing with reliability assessment. Independence between the observed variables is usually assumed; thus, the dependency between mechanical components is frequently overlooked.

Considering dependent relations between components is complex and prone to many uncertainties (An et al., 2016; Liu et al., 2012). A failure to consider this dependent relation will lead to imprecision and significant error when assessing the reliability of correlated mechanical components (Gu et al., 2022). The most traditional approach that considers dependent relations between variables only involves correlation coefficients in their reliability assessment models (Li et al., 2018; Yu & Wang, 2018). The disadvantages of these proposed methods are that correlation coefficients can only take linear dependency and are not flexible enough to take more than two variables. Monte Carlo (MC) simulation was also used to estimate the reliability of systems considering dependence structure (Bian et al., 2023; Stern et al., 2017). However, a more advanced technique, such as Finite Element Analysis (FEA), must be incorporated. Another method uses the Bayesian Network to model the correlations between variables. Bayesian Networks (BN) can construct the structure's functions and eliminate the independent assumption (Chen et al., 2021; Xiahou et al., 2023). In addition, Xiao et al. (2022) proposed a reliability assessment method by integrating Kriging predictions and parallel learning strategies. Although several studies have worked on the dependent issue, the usual proposed methods are not flexible enough. Linear relationships are usually assumed, and the methods cannot involve more than two variables. Thus, a more flexible and robust method is needed to capture the dependence structures between correlated variables.

Copula function is a mathematical approach combining two or more correlated variables by transforming the original data into copula domains. Copula functions have been widely used in many disciplines, from business and economics to science and engineering. Copula functions are flexible to model correlated variables as they do not require any univariate distribution function to be identified. Copula functions can also accommodate multivariate modeling involving more than two variables (Pranowo & Ramadhani, 2023; Ramadhani et al., 2021). Copula functions used to model correlated variables in reliability assessment are usually implemented in structural reliability assessment considering geotechnical and environmental loads acting on the observed structures (Amini et al., 2021; Ramadhani et al., 2022a, 2022b; Wang & Li, 2019; Wu, 2015). Thus, applying copula functions to capture any dependence structures between correlated variables needs further investigation for mechanical systems.

This paper aims to introduce copula functions to capture the dependent relation between mechanical components. The Archimedean copulas family is used to illustrate dependence modeling. Previous studies only focused on applying the normal copula, t copula and Clayton copula, which cannot describe all possible characteristics of data distribution. Archimedean copula families can model data that show heavy-tailed data in the upper and lower domains, and data that show no heavy-tailed pattern. A simplified case is selected to illustrate the copula-based reliability assessment. To compare the results, three scenarios are used to estimate the reliability of mechanical components: independent, copula-based, and weakest link.

Methodology

Introduction to Copula Functions

Sklar introduced copula functions back in 1959, where copula functions were able to combine two or more correlated variables. Copula functions can model joint probability distributions where their univariate marginal distributions were not the main concern (Nelsen, 2006). Sklar's Theorem mentioned that let H be the n-dimensional distribution functions with their univariate distribution $F_1, F_2, \dots, F_n$, then a copula C exists such that.

$$H(x_1, x_2, \dots, x_n) = C(F_1(x_1), F_2(x_2), \dots, F_n(x_n)) \quad \text{Eq. 1}$$

From Sklar's Theorem, it can be reiterated that copula functions do not require the univariate marginal distribution to be identified. The observed random variables are uniformly transformed in the copula domains on [0,1] (Nelsen, 2006). The transformation of real data into copula domain can be performed by following pseudo-observation that satisfies the following equation.

$$U_i = \frac{R_i}{n+1} = \frac{n\widehat{F_i}(X_i)}{n+1} \quad \text{Eq. 2}$$

Where R_i is the ranks of each data, and n is the number of observations, and $\widehat{F_i}$ is the empirical cumulative function.

Copula functions are then classified as

$$C(u_1, u_2, \dots, u_n) = P(U_1 \le u_1, U_2 \le u_2, \dots, U_n \le u_n) \quad \text{Eq. 3}$$

There are many copula functions to choose from. In this paper, the Archimedean copulas family is selected due to their flexibility to be reproduced. Out of all Archimedean copulas, Clayton, Gumbel, and Frank copulas are used to model the correlated variables in this paper. The probability distributions of these copulas for bivariate cases are presented in Table 1.

Table 1. Probability distributions for bivariate Archimedean copulas and their parameter (θ)

Copula	$C_\theta(u_1, u_2)$	$\theta \in$
Clayton	$\left(u_1^{-\theta} + u_2^{-\theta} - 1\right)^{-1/\theta}$	$(1, \infty)$
Gumbel	$\exp\left\{-\left[(-\ln u_1)^\theta + (-\ln u_2)^\theta\right]^{1/\theta}\right\}$	$[1, \infty)$
Frank	$-\frac{1}{\theta}\ln\left(1 + \frac{(e^{-u_1\theta} - 1)(e^{-u_2\theta} - 1)}{e^{-\theta} - 1}\right)$	$(-\infty, \infty)$

Copula-based Reliability of a Series System

The reliability of a series system is explained that if one component fails, the whole system will fail. The schematic diagram of the reliability of a series system is shown in Figure 1. The reliability of a mechanical system is often a function of time, number of cycles, milage, and others.

Figure 1. Reliability of a series system

Let a mechanical system be composed of n components in series, their life data variable $(T_1, T_2, \dots, T_n)$, and the joint probability distribution $H(t_1, t_2, \dots, t_n) = P\{T_1 \le t_1, T_2 \le t_2, \dots, T_n \le t_n\}$. For a series system, the reliability of the system is expressed by the smallest life of all units (An et al., 2016). The reliability of a series system consisting of two components is denoted as $R(t) = P\{\min(T_1, T_2) > t\} = P\{T_1 > t, T_2 > t\} = 1 - \left(F_1(t) + F_2(t)\right) + P(T_1 < t, T_2 < t)$

Substituting a copula to model the joint probability of $P(T_1 < t, T_2 < t)$, the copula-based reliability equation for a series system is then denoted as

$$R(t) = 1 - F_1(t) - F_2(t) + C(F_1(t), F_2(t); \theta) \quad \text{Eq. 4}$$

Copula-based Reliability of a Parallel System

For a parallel system, the failure of all components in the unit will lead to the failure of the system. The schematic diagram of a parallel system is shown in Figure 2. The biggest life of all units in a system expresses the reliability of a parallel system. The reliability of a parallel

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system consisting of two components is expressed as

$$R(t) = P\{\max(T_1, T_2) > t\}$$
$$= 1 - P\{T_1 < t, T_2 < t\}$$

Replacing the joint probability of $P\{T_1 < t, T_2 < t\}$ with a Copula will result in a copula-based reliability equation for a parallel system as follows.

$$R(t) = 1 - C(F_1(t), F_2(t); \theta) \quad \text{Eq. 5}$$

Figure 2. Reliability of a parallel system

Results and Discussions

A case study was selected to illustrate the application of copula functions to estimate the reliability of a mechanical system. In this paper, a mechanical system consisting of the crank and connecting rod is selected. These two components work as a series system. Data is generated based on a selected case study. The crank and connecting rod of a diesel engine are selected as the case in the paper, and their life distribution (in mileage) follows the lognormal and Weibull distribution, respectively (Gu et al., 2022). The distribution parameters of the life data of the crank and connecting rod are presented in Table 2.

Table 2. Distribution parameters of the crank and connecting rod

Component	Distribution	Shape parameter	Scale parameter	Mean	Standard deviation
Crank	Lognormal			10.90	1.23
Connecting rod	Weibull	4802 1.18	1.36		

Figure 3 shows the generated life distribution data between the crank and connecting rod correlates strongly. The data will scatter more if the two variables do not show any strong correlation. Thus, assuming independence between the life data of the crank and connecting rod will result in misinterpretation of the actual data trend. Clayton, Gumbel, and Frank Copulas are then used to model the bivariate dependence between the observed variables. The parameter of each copula is estimated, and the best copula model is determined by the Akaike Information Criterion (AIC) value. Compared to Mean Square Error (MSE), AIC gives a more accurate and stable result. MSE tends to converge to local extremes or even potentially diverge (Pranowo & Ramadhani, 2023). In addition, AIC is also more practical in engineering application. Table 3 shows the parameters for each copula along with their AIC values, while Figure 4 shows the probability distribution of the fitted copulas to the life data.

Table 3. Copula parameters and their AIC values

Copula	Parameter (θ)	AIC
Clayton	1.0064	190.4097*
Gumbel	1.7077	253.7201
Frank	-4.4404	223.0998

**Indicates the best copula model*

From Table 3, the copula parameter is estimated using the maximum loglikelihood estimation (MLE) method. Table 3 also shows that Clayton has the smallest AIC value, indicating the best-fitted copula to model the correlated bivariate case of the observed variables. This result is also validated in Figure 4. Clayton copula tends to generate correlated data showing a heavy tail in the lower bound area, as seen in Figure 3. Thus, the Clayton copula is selected to model the dependence between the connecting rod and crank to estimate the reliability between these two components.

After investigating the dependence between the observed variables and the best copula model is selected, the system's reliability is then estimated Thus, Equation 3 is used to estimate the reliability of a series system. Three scenarios are presented to investigate the advantage of using copula functions to estimate the system's reliability. Figure 5 shows the estimated reliability of the crank and connecting rod system, considering three different scenarios. The first scenario is that the relationship between the crank and connecting rod is independent. The second scenario

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considers the dependent relation between two variables modeled using the copula functions. The last scenario, considering the system's reliability, is estimated using the weakest link theory. This theory explains that in every system, there must be at least one variable that limits the performance of the system. In this paper, the weakest link is the component that has the smallest value of component reliability. Figure 5 shows that reliability estimated using Clayton copula results in higher values compared to the independent case and the weakest link theory. This result is because copula-based reliability estimation considers adding the copula value to describe the relationship between two correlated variables. Thus, the ignorance of this dependence will result in an inaccurate estimation of the system’s reliability. The reliability of the weakest link acts as the upper bound in this comparison. Thus, the estimated copula-based reliability is still between the upper and lower bounds. These results satisfy the boundary theory of reliability, which states that the lower bound of a system’s reliability is estimated from the assumption of an independent case. While the upper bound is obtained from the weakest link (Gu et al., 2022; Tang et al., 2013). Figure 6 shows the comparison of the estimated reliability modelled using three different Archimedean copula functions. From this figure, the best fitted copula: Clayton function, results in slightly different result and trend compared to the other two copulas in higher mileage. This is because the main characteristic of Clayton copula is that it generates heavy-tailed data in lower domain. All in all, incorporating copula functions to estimate the reliability of a mechanical system is effective and practical.

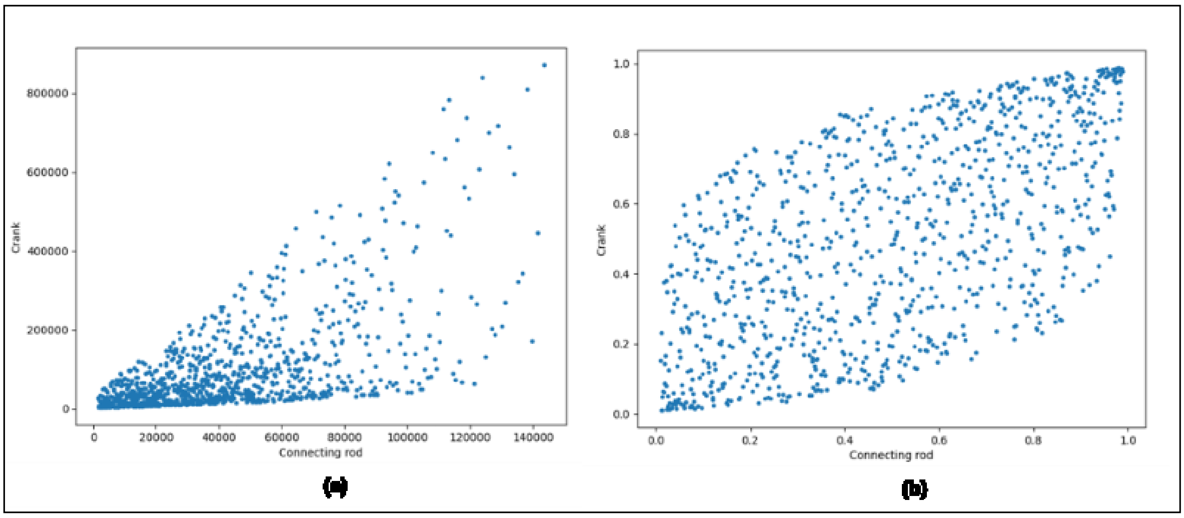

Figure 3. Scatter plots for the life distribution between crank and connecting rod in (a) original domain, (b) copula domain.

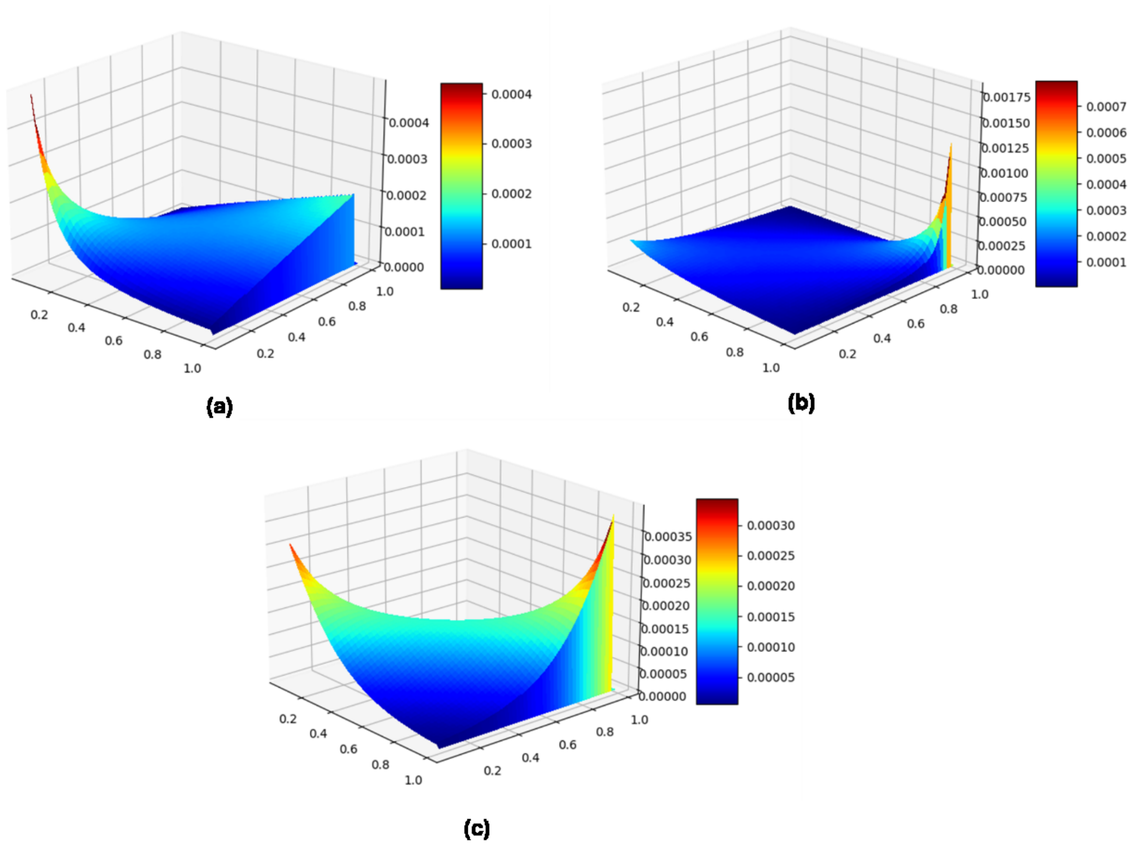

Figure 4. Probability distribution of (a) Clayton, (b) Gumbel, (c) Frank copulas

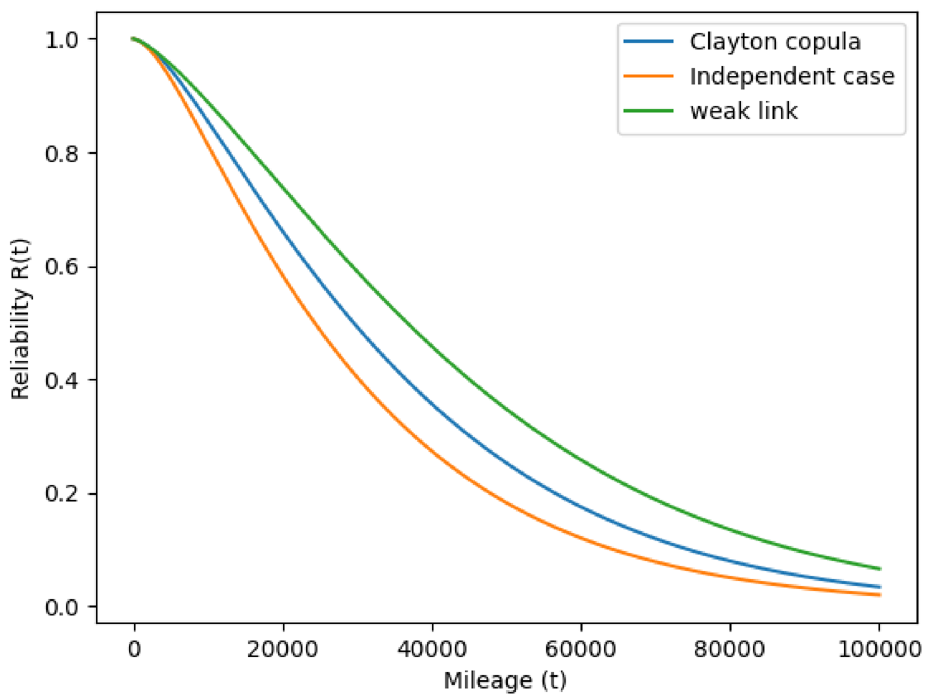

Figure 5. The estimated reliability of a series system considering three different scenarios

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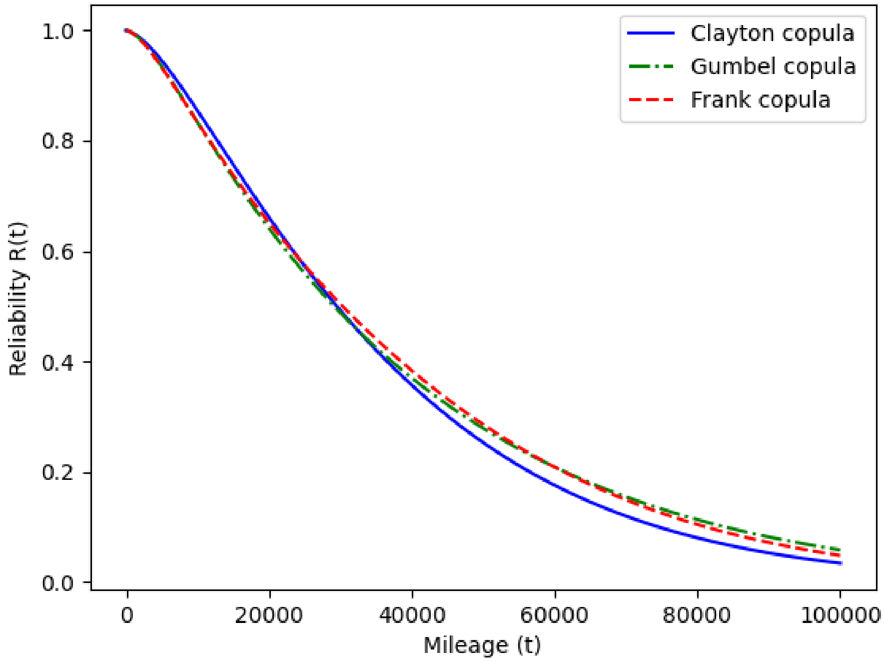

Figure 6. The estimated reliability of a series system considering different copula functions

Conclusion

The Archimedean copulas family is introduced to propose a copula-based reliability estimation for a mechanical system. Clayton, Gumbel, and Frank copula are selected to model the dependence between the correlated crank and connecting rod. The generated life data of the crank and connecting rod show a strong correlation, which shows that the independent assumption to calculate the system's reliability is irrelevant. From the fitting, the Clayton copula is the best copula to capture the dependence of the life data between the crank and connecting rod. The reliability of the series system between the crank and connecting rod is carried out by incorporating the Clayton copula into the model. Reliability estimation using copula provides more relevant and reasonable results. The reliability of the series system obtained using a copula can satisfy the boundary theory of reliability. The copula-based reliability values are between the reliability for the weakest link and independent-based reliability, which act as the upper and lower bound, respectively. Incorporating copula functions to estimate mechanical system reliability provides new insights and perspectives on eliminating the assumption of independence between the observed variables. The proposed method can be implemented to model any dependence structures found in identifying failure modes in estimating the reliability of mechanical systems. Future research includes the study of copula-based reliability estimation for parallel mechanical systems, considering different failure modes in a mechanical system, and using different copula families.

References

Amini, A., Abdollahi, A., Hariri-Ardebili, M. A., & Lall, U. (2021). Copula-based reliability and sensitivity analysis of aging dams: Adaptive Kriging and polynomial chaos Kriging methods. Applied Soft Computing, 109, 107524. https://doi.org/10.1016/j.asoc.2021.107524

An, H., Yin, H., & He, F. (2016). Analysis and Application of Mechanical System Reliability Model Based on Copula Function. Polish Maritime Research, 23(s1), 187–191. https://doi.org/10.1515/pomr-2016-0064

Aritonang, Y. M. K. (2023). Pengembangan Pendekatan Matematis dan Penyelesaiannya untuk Reliabilitas Sistem Dengan Distribusi Kegagalan Berbeda.

Jurnal Rekayasa Sistem Industri, 12(1), 35–42. https://doi.org/10.26593/jrsi.v12i1.6318.35-42

Avontuur, G. C., & van der Werff, K. (2002). Systems reliability analysis of mechanical and hydraulic drive systems. Reliability Engineering & System Safety, 77(2), 121–130. https://doi.org/10.1016/S0951-8320(02)00039-X

Bian, L., Wang, G., & Liu, P. (2023). Reliability analysis for <math altimg="si9.svg" display="inline" id="d1e1612"> <mi>k</mi> </math> -out-of- <math altimg="si10.svg" display="inline" id="d1e1617"> <mi>n</mi> </math> (G) systems subject to dependent competing failure processes. Computers & Industrial Engineering, 177, 109084. https://doi.org/10.1016/j.cie.2023.109084

Chen, R., Zhang, C., Wang, S., & Qian, Y. (2021). Reliability estimation of mechanical seals based on bivariate dependence analysis and considering model uncertainty. Chinese Journal of Aeronautics, 34(5), 554–572. https://doi.org/10.1016/j.cja.2020.12.001

de Reffye, J., & Dersin, P. (2010). Mechanical Reliability of a point system. IFAC Proceedings Volumes, 43(3), 86–91. https://doi.org/10.3182/20100701-2-PT-4012.00016

Feizabadi, M., & Jahromi, A. E. (2017). A new model for reliability optimization of series-parallel systems with non-homogeneous components. Reliability Engineering & System Safety, 157, 101–112. https://doi.org/10.1016/j.ress.2016.08.023

Gu, Y.-K., Fan, C.-J., Liang, L.-Q., & Zhang, J. (2022). Reliability calculation method based on the Copula function for mechanical systems with dependent failure. Annals of Operations Research, 311(1), 99–116. https://doi.org/10.1007/s10479-019-03202-5

Hu, Y., Ding, Y., Wen, F., & Liu, L. (2019). Reliability Assessment in Distributed Multi-State Series-Parallel Systems. Energy Procedia, 159, 104–110. https://doi.org/10.1016/j.egypro.2018.12.026

Jiang, D., Chen, T., Xie, J., Cui, W., & Song, B. (2023). A mechanical system reliability degradation analysis and remaining life estimation method——With the example of an aircraft hatch lock mechanism. Reliability Engineering & System Safety, 230, 108922. https://doi.org/10.1016/j.ress.2022.108922

Li, H., Huang, H.-Z., Li, Y.-F., Zhou, J., & Mi, J. (2018). Physics of failure-based reliability prediction of turbine blades using multi-source information fusion. Applied Soft Computing, 72, 624–635. https://doi.org/10.1016/j.asoc.2018.05.015

Li, Y., Xia, T., Chen, Z., & Pan, E. (2023). Multiple degradation-driven preventive maintenance policy for serial-parallel multi-station manufacturing systems. Reliability Engineering & System Safety, 230, 108905. https://doi.org/10.1016/j.ress.2022.108905

Liu, Z., Tao, F. H., & Jia, C. Z. (2012). Application of Copula in Reliability Theory. Applied Mechanics and Materials, 217–219, 2746–2749. https://doi.org/10.4028/www.scientific.net/AMM.217-219.2746

Louzada, F., Tomazella, V. L. D., Gonzatto, O. A., Bochio, G., Milani, E. A., Ferreira, P. H., & Ramos, P. L. (2022). Reliability assessment of repairable systems with series–parallel structure subjected to hierarchical competing risks under minimal repair regime. Reliability Engineering & System Safety, 222, 108364. https://doi.org/10.1016/j.ress.2022.108364

Maukar, A. L., Sosodoro, I. W., & Adiprabowo, R. (2016). Scheduling Preventive Maintenance on Auto Rooting Machine at Toys Manufacturer Company. Jurnal Rekayasa Sistem Industri, 5(1), 26. https://doi.org/10.26593/jrsi.v5i1.1910.26-30

Mustaqim, F., Kosasih, W., & Ahnad, A. (2020). Pemeliharaan Mesin Hydraulic Shear Menggunakan Pendekatan Reliability Centered Maintenance dan Manajemen Suku Cadang. Jurnal Rekayasa Sistem Industri, 9(3), 153–162. https://doi.org/10.26593/jrsi.v9i3.4023.153-162

Nelsen, R. B. (2006). An Introduction to Copulas. Springer New York. https://doi.org/10.1007/0-387-28678-0

Pranowo, W., & Ramadhani, A. R. (2023). A Comparison of Python-Based Copula Parameter Estimation for Archimedean-Based Asymmetric Copulas. SN Computer Science, 4(2), 207.

https://doi.org/10.1007/s42979-023-01674-8
Ramadhani, A., Khan, F., Colbourne, B., Ahmed, S., & Taleb-Berrouane, M. (2021). Environmental load estimation for offshore structures considering parametric dependencies. Safety in Extreme Environments. https://doi.org/10.1007/s42797-021-00028-y
Ramadhani, A., Khan, F., Colbourne, B., Ahmed, S., & Taleb-Berrouane, M. (2022a). Resilience assessment of offshore structures subjected to ice load considering complex dependencies. Reliability Engineering & System Safety, 222, 108421. https://doi.org/10.1016/j.ress.2022.108421
Ramadhani, A., Khan, F., Colbourne, B., Ahmed, S., & Taleb-Berrouane, M. (2022b). Resilience assessment of offshore structures subjected to ice load considering complex dependencies. Reliability Engineering & System Safety, 222, 108421. https://doi.org/10.1016/j.ress.2022.108421
Stern, R. E., Song, J., & Work, D. B. (2017). Accelerated Monte Carlo system reliability analysis through machine-learning-based surrogate models of network connectivity. Reliability Engineering & System Safety, 164, 1–9. https://doi.org/10.1016/j.ress.2017.01.021
Tang, X.-S., Li, D.-Q., Zhou, C.-B., & Zhang, L.-M. (2013). Bivariate distribution models using copulas for reliability analysis. Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability, 227(5), 499–512. https://doi.org/10.1177/1748006X13481928
Wang, F., & Li, H. (2019). A non-parametric copula approach to dependence modelling of shear strength parameters and its implications for geotechnical reliability under incomplete probability information. Computers and Geotechnics, 116, 103185. https://doi.org/10.1016/j.compgeo.2019.103185
Wei, Y., & Liu, S. (2023). Reliability analysis of series and parallel systems with heterogeneous components under random shock environment. Computers & Industrial Engineering, 179, 109214. https://doi.org/10.1016/j.cie.2023.109214
Woo, S., & O'Neal, D. L. (2019). Reliability design and case study of mechanical system like a hinge kit system in refrigerator subjected to repetitive stresses. Engineering Failure Analysis, 99, 319–329. https://doi.org/10.1016/j.engfailanal.2019.02.015
Wu, X. Z. (2015). Modelling dependence structures of soil shear strength data with bivariate copulas and applications to geotechnical reliability analysis. Soils and Foundations, 55(5), 1243–1258. https://doi.org/10.1016/j.sandf.2015.09.023
Xiahou, T., Zheng, Y.-X., Liu, Y., & Chen, H. (2023). Reliability modeling of modular k-out-of-n systems with functional dependency: A case study of radar transmitter systems. Reliability Engineering & System Safety, 233, 109120. https://doi.org/10.1016/j.ress.2023.109120
Xiao, N.-C., Yuan, K., & Zhan, H. (2022). System reliability analysis based on dependent Kriging predictions and parallel learning strategy. Reliability Engineering & System Safety, 218, 108083. https://doi.org/10.1016/j.ress.2021.108083
Yu, S., & Wang, Z. (2018). A Novel Time-Variant Reliability Analysis Method Based on Failure Processes Decomposition for Dynamic Uncertain Structures. Journal of Mechanical Design, 140(5). https://doi.org/10.1115/1.4039387

DOI: https://doi.org/10.26593/jrsi.v13i2.7219.103-112

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