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ANALISA OPTIMASI PORTOFOLIO YANG MEMUAT SAHAM-SAHAM KELOMPOK LQ45

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DAFTAR ISI

Analysis of Portfolio Optimization Consisting of Stocks in LQ45 Index

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Abstract. Portfolio optimization is the process of choosing the proportions of various assets to be held in a portfolio in such way as to make the portfolio better than any other according to some criteria, e.g. minimize the risk and/or maximize the return. In this article, we will determine the proportions of each stock that optimizes the portfolio consisting of stocks in LQ45 index. We analyse the riskiest sector in LQ45 index, as well. LQ45 index was launched in February 1997. This index using 45 selected stocks with criteria specified by Indonesia Stock Exchange, among which are liquidity and market capitalization. The optimum criteria used in the portfolio are to minimize the risk with and without a particular return target. For both these criteria, short-selling is not allowed. We use Newton method to determine proportions that optimize the portfolio. We find that a portfolio that contains stocks from agriculture sector is the riskiest sector in LQ45 index whereas a portfolio that contains stocks from property sector gives the highest return.

1. Introduction

Portfolio is defined as a collection of investment that consisting of various types of assets, such as bonds, stocks, and cash. Portfolio optimization is the process of choosing the proportions of various assets to be held in a portfolio in such way as to make the portfolio better than any other according to some criteria, e.g. minimize the risk and/or maximize the return.

LQ45 index, which was launched in February 1997, is one of the indicators of stocks in the capital market in Indonesia. LQ45 index using the 45 stocks that selected by the Indonesia Stock Exchange with some criteria, i.e. liquidity and market capitalization. According to [1], transaction value in the regular market is the basic measure of liquidity, and since January 2005, number of trading days and frequency of transactions is also used as a measure of liquidity. Stocks of the index LQ45 will be evaluated every 3 months and the stocks of index LQ45 can be changed every six months, i.e. in the beginning of February and August.

In [2], Biggs discuss how to find minimum the risk of portfolio when short-selling is not permitted with a particular return target. In this article, we want to apply that model to stocks in LQ45 index, as well as the model to find the minimum risk of portfolio when short-selling is not permitted without a particular return target. We choose a number of stocks in LQ45 based on the best average return and use those models to determine the invested proportions of the stocks that optimize the portfolios. We also calculate the minimum risk and return of portfolios (only for case without a particular return target). Besides that, we also give some results if we choose some stocks from 9 sectors (agriculture, mining, property, consumer goods, basic industry, miscellaneous industry, finance, infrastructure, and trading) or based on market capitalization in LQ45 index.

2. Portfolio Optimization

2.1. The Model

Suppose that we have n assets in a particular portfolio and r_{ij} is history of returns (in percent) from asset i on period j with $i = 1,2,\dots,n$, $j = 1,2,\dots,m$, and assume that $m > n$. Let y_i $(i = 1,2,\dots,n)$ is

invested proportions for asset i and definitely that $\sum_{i=1}^{n} y_i = 1$. The risk of portfolio can be defined as the variance of the portfolio and can be written as [2]

$$V = \boldsymbol{y}^T Q \boldsymbol{y} \tag{1}$$

where

$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \text{ and } Q = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn} \end{pmatrix}$$

Matrix Q is called a variance-covariance matrix. The variance of asset i ($\sigma_{ii} = \sigma_i^2$) and the covariance of assets i and k (σ_{ij}), respectively can be calculated from the history of returns r_{ij}, that is

$$\sigma_{ii} = \frac{1}{m} \sum_{j=1}^{m} \left(r_{ij} - \bar{r}_i \right)^2$$

and

$$\sigma_{ij} = \frac{1}{m} \sum_{k=1}^{m} \left(r_{ik} - \bar{r}_i \right) \left(r_{jk} - \bar{r}_j \right)$$

where $\bar{r}_i$ is mean return of asset i.

This article will discuss the number of selected stocks and the invested proportions y_i for each stock that optimizes the portfolio. The optimum criteria used in the portfolio are to minimize the risk V with and without a particular return target. For both these criteria, short-selling is not allowed, so that $y_i \geq 0$ for every i.

As stated in (1), so the problem of minimize the risk of portfolio consisting of n assets can be written as

$$\text{Min } V = \boldsymbol{y}^T Q \boldsymbol{y} = \sum_{i=1}^{n} \sigma_i^2 \, y_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sigma_{ij} \, y_i y_j \tag{2}$$

with respect to $\sum_{i=1}^{n} y_i = 1$ and $y_i \geq 0$ for every i. Optimization problem with constraints in (2) can be transform to unconstraint one using penalty function and change y_i to x_i^2 (this change ensures that the invested proportions y_i are non-negative). So, (2) become

$$\text{Min } F(\boldsymbol{x}) = \sum_{i=1}^{n} x_i^2 \left(\sum_{j=1}^{n} \sigma_{ij} x_j^2 \right) + \rho \left(\sum_{i=1}^{n} x_i^2 - 1 \right)^2 \tag{3}$$

where ρ is a penalty parameter.

Whereas, the problem of minimize the risk of portfolio consisting of n assets with a particular return target R_p can be written as [2]

$$\text{Min } V = \boldsymbol{y}^T Q \boldsymbol{y} = \sum_{i=1}^{n} \sigma_i^2 y_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \sigma_{ij} y_i y_j \tag{4}$$

with respect to $\sum_{i=1}^{n} \bar{r}_i y_i = R_p$, $\sum_{i=1}^{n} y_i = 1$, and $y_i \geq 0$ for every i. Using the same way as we stated in (3), we can transform (4) become to [2]

$$\text{Min } F(\boldsymbol{x}) = \sum_{i=1}^{n} x_i^2 \left(\sum_{j=1}^{n} \sigma_{ij} x_j^2 \right) + \rho \left(\sum_{i=1}^{n} x_i^2 - 1 \right)^2 + \bar{\rho} \left[\sum_{i=1}^{n} \bar{r}_i x_i^2 - R_p \right]^2 \tag{5}$$

where $\bar{\rho} = \frac{\rho}{R_p^2}$.

2.2. How to Solve the Problems?

In previous subsection, we see that the problem of minimize the risk of portfolio consisting of *n* assets without a particular return target, can be transform to unconstraint problem and given as

$$\text{Min } F(\boldsymbol{x}) = \sum_{i=1}^{n} x_i^2 \left(\sum_{j=1}^{n} \sigma_{ij} x_j^2 \right) + \rho \left(\sum_{i=1}^{n} x_i^2 - 1 \right)^2 \tag{6}$$

The first derivative of objective function F in (6) with respect to x_i for every i is given by

$$\frac{\partial F}{\partial x_i} = 4x_i \left[\rho \left(\sum_{i=1}^{n} x_i^2 - 1 \right) + 2 \sum_{k=1}^{n} \sigma_{ik} x_k^2 \right] \tag{7}$$

Similarly, the problem of minimize the risk of portfolio consisting of *n* assets with a particular return target R_p can be transform to unconstraint problem and given as

$$\text{Min } F(\boldsymbol{x}) = \sum_{i=1}^{n} x_i^2 \left(\sum_{j=1}^{n} \sigma_{ij} x_j^2 \right) + \rho \left(\sum_{i=1}^{n} x_i^2 - 1 \right)^2 + \bar{\rho} \left[\sum_{i=1}^{n} \bar{r}_i x_i^2 - R_p \right]^2 \tag{8}$$

The first derivative of objective function F in (8) with respect to x_i for every i is given by [1]

$$\frac{\partial F}{\partial x_i} = 4x_i \left[\bar{\rho} \left(\sum_{k=1}^{n} \bar{r}_k x_k^2 - R_p \right) \bar{r}_i + \rho \left(\sum_{i=1}^{n} x_i^2 - 1 \right) + 2 \sum_{k=1}^{n} \sigma_{ik} x_k^2 \right] \tag{9}$$

Both of (7) and (9) is equal to zero and we will find x_i for every i, that is, we will find the critical points of function F. We are applied Newton method to solve these nonlinear equation systems. After we get the x_i, so we can determine the invested proportions y_i by squared the x_i for every i.

3. The Results

3.1. Minimize the Risk of Portfolio without a Particular Return Target

In this article, we will determine the number of selected stocks in LQ45 index and the invested proportions y_i for each stock that optimizes the portfolio. The stocks that we choose in portfolio are determined by ordering the average of return of each stock from highest to lowest. We use the history price of stocks in yahoo finance [3] from 1 August 2014 to 31 July 2015 that consist of 247 trading day. From the data, we calculated the average of return of each stock and ordering them from highest to lowest.

Let an investor choose 5 stocks in LQ45 index with the best average return in his portfolio, i.e. PT. Waskita Karya Tbk. (WSKT), PT. Pembangunan Perumahan Tbk. (PTPP), PT. Sawit Sumbermas Sarana Tbk. (SSMS), PT. Summarecon Agung Tbk. (SMRA), and PT. Unilever Indonesia Tbk. (UNVR). He wants to minimize the risk of portfolio and short-selling is not permitted. Then, the proportions of each stock are given in table 1.

Table 1. Proportions of 5 stocks in a portfolio with the minimum risk and return.

WSKT	PTPP	SSMS	SMRA	UNVR	Min. risk	Return (%)
0.1402	0.1630	0.1686	0.0883	0.4368	1.5676	0.1843

In table 1, we can see each proportion of stocks in his portfolio. The return that he expected is 0.1843% with the minimum risk is 1.5676.

In table 2, we want to show the minimum risk and return of portfolios that consist of 5 stocks, 10 stocks, until 45 stocks which given the best average of return, respectively. From table 2, we can see that the risk of portfolio become smaller as the number of stocks become larger. So, diversification in investment will minimize the risk of this portfolio and also the return.

Table 2. The minimum risk and return of portfolio that consist of a number of stocks.

Number of stocks in portfolio	Number of stocks with a proportion greater than zero	Minimum risk	Return (%)
5	5	1.5676	0.1843
10	8	0.9470	0.1383
15	11	0.7045	0.0951
20	14	0.6181	0.0750
25	16	0.6169	0.0729
30	17	0.6021	0.0580
35	17	0.6021	0.0580
40	17	0.5887	0.0396
45	20	0.5555	−0.0145

In table 2, we see that only portfolio which consist of 5 stocks give the invested proportions of each stock is greater than zero. So, if we make a particular portfolio which consists of 20 stocks (for example), we only purchase 14 stocks to get the minimum risk with expected return at 0.0750%.

The stocks are traded on Indonesia Stock Exchange can be grouped in 9 sectors, i.e., agriculture, mining, property, consumer goods, basic industry, miscellaneous industry, finance, infrastructure, and trading. Next, we make 9 portfolios. Each portfolio contain 5 stocks from each sector (except agriculture and basic industry sectors that contain only 3 stocks, respectively and miscellaneous industry sector that contain only 1 stock) in LQ45 index with the best average of return.

Table 3. The minimum risk and return of portfolios that consist of 5 stocks from each sector (except agriculture, basic industry, and miscellaneous industry sectors).

Sectors	Min. risk	Return (%)
Agriculture [a]	3.3405	0.0231
Mining	1.9813	−0.3301
Property	2.5528	0.1813
Consumer Goods	1.0782	0.0179
Basic Industry [a]	2.1983	−0.1364
Miscellaneous Industry [b]	2.7949	−0.0555
Finance	1.1901	0.0349
Infrastructure	0.8671	−0.0259
Trading	1.2598	0.0547

[a] Only 3 stocks
[b] Only 1 stock

In table 3, we see that portfolio which consists of 3 stocks from agriculture sector has the highest risk with expected return 0.0231%. The highest return of portfolio is given by a portfolio which consists of 5 stocks from property sector. So, if you an aggressive investor, then you can make a portfolio that contain 5 stocks from property sector. If you do not want to take the risk too high, then you can make a portfolio which contains 5 stocks from trading sector, because the expected return of this portfolio is higher than portfolios which consist of agriculture, consumer goods, or finance sectors and the risk is lower than portfolios which consist of agriculture or property sectors. (We do not compare with mining, basic industry, miscellaneous industry or infrastructure sectors because these sectors give a negative return if we make portfolios which consist of stocks from these sectors. Of course, an investor always expects the growing portfolio). In fact, the property and trading sector have 10 stocks and 8 stocks, respectively. If we make a portfolio which contain all stocks in property sector, then we

get the minimum risk is 2.3610 with expected return 0.1462%, and there is one stock with zero invested proportion. And, if we make a portfolio which contain all stocks in trading sector, then we have the minimum risk is 1.1608 with expected return 0.0327%.

Beside grouped in sector, we also grouped the stocks of LQ45 in market capitalization. Market capitalization is total market value of all of company's outstanding stocks. This value is calculated by multiplying a company's stocks outstanding by the current market price of one stock. We group the stocks in LQ45 index into 3 groups based on market capitalization, that is, Group 1 is the company with market capitalization larger than 100 trillion Rupiahs (9 stocks), Group 2 is the company with market capitalization between 20 and 100 trillion Rupiahs (24 stocks), and Group 3 is the company with market capitalization under 20 trillion Rupiahs (12 stocks). The data of market capitalization is obtained from [4]. Then, we make 3 portfolios. Each portfolio contains 8 stocks from each group with the best average of return and we minimize the risk of these portfolios. Here are the results.

Table 4. The minimum risk and return of portfolios that consist of 8 stocks from each group.

Group	Min. risk	Return (%)
1	0.9051	0.0391
2	1.3927	0.0474
3	1.0822	0.1349

In table 4, we see that the portfolio which consists of 8 stocks from Group 1 has the lowest minimum risk whereas the portfolio which consists of 8 stocks from Group 3 has the highest expected return. So, if you an investor, you can make a portfolio which consist of 8 stocks from Group 3 because it has the highest expected return and the risk in the middle.

3.2. Minimize the Risk of Portfolio with a Particular Return Target

In this subsection, we discuss about minimize the risk of portfolio with a specified return R_p in similar way as previous subsection. In the table below, we want to show the minimum risk of portfolio which contains 5 stocks that given the best average return with various of R_p. We also show the invested proportions from each stock.

Table 5. Proportions of 5 stocks in a portfolio with the minimum risk and various of R_p.

	Proportions					
	$R_p = 0.14\%$	$R_p = 0.16\%$	$R_p = 0.1843\%$	$R_p = 0.20\%$	$R_p = 0.22\%$	$R_p = 0.24\%$
WSKT	0	0.0352	0.1403	0.2081	0.2941	0.3829
PTPP	0.0990	0.1784	0.1629	0.1528	0.1401	0.1200
SSMS	0.1270	0.1537	0.1686	0.1782	0.1904	0.2005
SMRA	0.1873	0.1337	0.0883	0.0590	0.0218	0
UNVR	0.5815	0.4951	0.4368	0.3992	0.3515	0.2953
Minimum risk	1.7474	1.6148	1.5676	1.5897	1.6772	1.8317

In table 5 above, we see that if a fix of target value for return is given and become smaller, then the minimum risk is not become smaller. Figure 1 below shows the curve of return versus risk of a portfolio which consists of 5 stocks. In the figure, we can see that the curve is parabolic rather than linear. The curve is known as efficient Frontier curve.

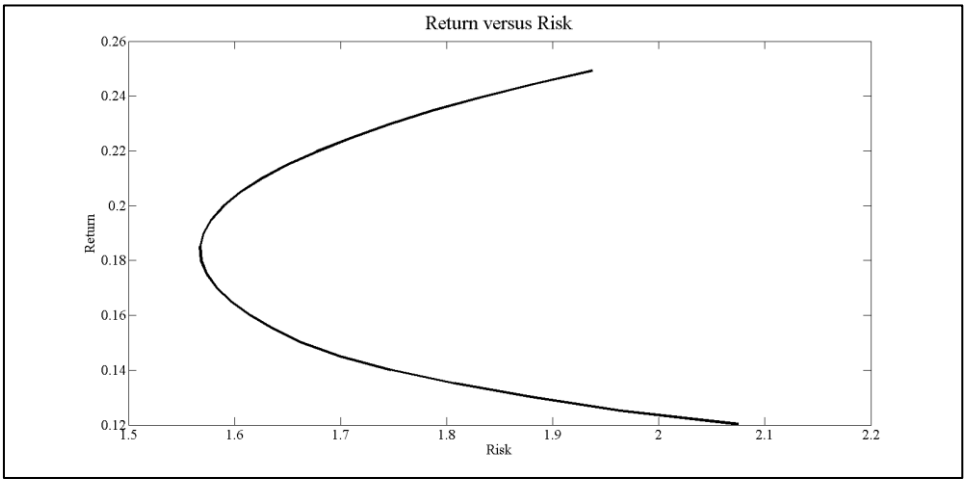

Figure 1. Curve of return versus risk of a portfolio which consists of 5 stocks

Next, we make 9 portfolios like we discuss in Table 3. However, if we make portfolios that contain stocks from mining, basic industry, miscellaneous industry or infrastructure sectors, the portfolios give a negative return. Definitely, an investor does not like this, so we only compare of 5 portfolios. The results give in table 6.

Table 6. The minimum risk of portfolios that consist of 5 stocks from property, consumer goods, finance or trading sectors (except agriculture sector which only consist of 3 stocks) with a specified return R_p.

	Minimum risk				
Sectors	$R_p = 0.05\%$	$R_p = 0.075\%$	$R_p = 0.1\%$	$R_p = 0.125\%$	$R_p = 0.15\%$
Agriculture [d]	3.3866	3.5115	3.7223	4.0654	4.5472
Property	4.8231	3.5444	3.0544	2.7445	2.6047
Consumer Goods	1.1840	1.4223	1.9959	*	*
Finance	1.2549	*	*	*	*
Trading	1.2656	1.3768	1.9393	*	*

[d] Only 3 stocks

In table 6, only portfolios that contain stocks from agriculture or property sectors give the feasible solutions for R_p from 0.05% up to 0.15% (asterisks sign mean the optimization problem does not give a feasible solution). The portfolio which contains 5 stocks from property sector can give the highest target of return than other sectors with lower risk than agriculture sector.

The last table, we make 3 portfolios that consist of 8 stocks from each group with the best average of return (the same group as we discuss on table 4). Then, we determine the minimum risk of these portfolios with various of return target R_p.

Table 7. The minimum risk of portfolios that consist of 8 stocks from each group with various of specified return R_p.

	Minimum risk					
Group	$R_p = 0.05\%$	$R_p = 0.06\%$	$R_p = 0.07\%$	$R_p = 0.08\%$	$R_p = 0.09\%$	$R_p = 0.1\%$
1	0.9244	0.9782	1.0742	1.2678	1.5894	2.0391
2	1.3954	1.4418	1.5490	1.7210	1.9661	2.2920
3	1.7785	1.5078	1.3577	1.2587	1.1931	1.1442

In table 7, an investor can make a portfolio which consists of 8 stocks from Group 3 because this portfolio gives the lowest risk than others when the specified return is 0.1%.

4. Conclusions

If we use data LQ45 from 1 August 2014 until 31 July 2015, we can see that the portfolio which consists of stocks from agriculture sector is the highest risk whereas the portfolio which consists of stocks from property sector gives the highest return. A moderate investor can make a portfolio that consist of 8 stocks with the best average of return from company's stocks with market capitalization is under 20 trillion Rupiahs.

5. References

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